Indian Olympiad Qualifier IOQA (2020 - 21) Solution

1. $\tan^{-1}\left[\frac{\cos x}{1-\sin x}\right]=\tan^{-1}\left[\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{1-2\sin\frac{x}{2}\cos\frac{x}{2}}\right]$

$$=\tan^{-1}\left[\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}\right]$$

$$=\tan^{-1}\left[\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{2}{2}\right)^2}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}\right]=\tan^{-1}\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}$$

$$=\tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]=\tan^{-1}\left[\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}\right]=\tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)=\frac{x}{2}+\frac{\pi}{4}$$

Ans: d

2. $\begin{vmatrix}1 & 1 & -1\\ -1 & 1 & 1\\ 1 & 1 & x\end{vmatrix}=0\Rightarrow 1\times(x-1)+1\left(1-(-x)\right)+(-1)(-1-1)=x-1+1+x+2=2x+2=0$

$$\Rightarrow x=-1$$

Ans: c

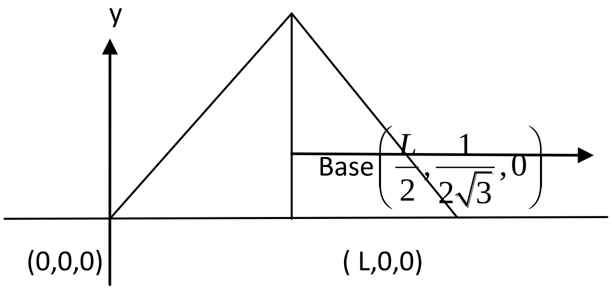
3. $\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\frac{dx}{Sin^2x\ Cos^2x}=\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\left[\frac{1}{Sin^2x}+\frac{1}{Cos^2x}\right]dx$

$$=\left[\tan x-\cot x\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}}=\left[(1-1)\right]-\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)=-\frac{2}{\sqrt{3}}$$

Ans: d

4. The points A, B, C are

$$\left(\frac{L}{2},\frac{L\sqrt{3}}{2},0\right)$$



If D is at $\left(\frac{L}{2},\frac{L}{\sqrt{2}},Z\right)$ then $\frac{L^2}{4}+\frac{L^2}{12}+Z^2=L^2$

$or \quad Z^2=\frac{2L^2}{3}\Rightarrow Z=\pm L\sqrt{\frac{2}{3}}$ Thus D is at $\left[\frac{L}{2},\frac{L}{2\sqrt{3}},\pm L\sqrt{\frac{2}{3}}\right]$

Ans: b

5. $Z=\frac{1+i}{1-i\sqrt{3}}=\frac{\sqrt{2}\,e^{i\pi/4}}{2e^{-i\pi/3}}=\frac{1}{\sqrt{2}}e^{i\left(\frac{\pi}{4}+\frac{\pi}{3}\right)}=\frac{1}{\sqrt{2}}e^{i\left(\frac{7\pi}{4}\right)}\because\frac{\pi}{4}+\frac{\pi}{3}=\frac{7\pi}{4}$ is the argument

Ans: b

6. $y_1=\frac{4x^2}{\pi^2}$ $y_2=sin\,x$ At the point of intercept $sin\,x=\frac{4x^2}{\pi^2}\Rightarrow x=\frac{\pi}{2}$ for $0<x<\pi$

Now $\left.\frac{dy_1}{dx}\right|_{\frac{\pi}{2}}=\frac{8x}{\pi^2}=\frac{8}{\pi^2}\frac{\pi}{2}=\frac{4}{\pi}=\tan\theta_1$

$\left|\left.\frac{dy_2}{dx}\right|_{\frac{\pi}{2}}\right.=cos\,x|_{\frac{\pi}{2}}=cos\frac{\pi}{2}=0\Rightarrow\tan\theta_2=0\Rightarrow\theta_2=0$ At the point,the second curve is parallel to x axis hence the angle between the two curves shall be $\theta=\tan^{-1}\frac{4}{\pi}$

Ans: b

7. For the three points to be collinear

$$\begin{vmatrix}2 & 3 & -4\\ 1 & -2 & 3\\ 3 & 8 & r\end{vmatrix}=0$$

$$\Rightarrow 2(-2r-24)-3(r-9)-4(8+6)=0$$

$or -4r-3r-48+27-56=0 \quad\Rightarrow r=-11$

Ans:c

8. Time measured $= n\pi \pm \frac{1}{100} = nT$

Or $T = \pi \pm \frac{1}{100n} = \pi\left[1 \pm \frac{1}{n\pi \times 100}\right]$

Therefore % error $= \frac{1}{n\pi \times 100} \times 100 = \frac{1}{n\pi} = (n\pi)^{-1}$ %

Ans: a

9. Force =mass x acceleration $8 \times 12 = 96N$

Ans: b

10. Using $v^2 - u^2 = 2as$; Distance travelled $= \frac{400}{2 \times a}$; $ma = \frac{1}{4}a = 30 \Rightarrow a = 120$

Therefore distance travelled $= \frac{400}{2 \times 120}$ Therefore the Work done $= \frac{400}{2 \times 120} \times 30 = 50J$

Alternately $\frac{1}{2}mv^2 = \frac{1}{2} \times \frac{1}{4} \times 20^2 = 50J$

Ans: b

11. The equation of SHM may be $x = x_0 \sin \omega t$ where $\omega = \sqrt{\frac{k}{m}}$ and

Now the Kinetic Energy is $KE = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\omega^2 x_0^2 \cos^2 \omega t$ Thus $KE_{\max} = \frac{1}{2}m\omega^2 x_0^2$

$$Power = \frac{dE}{dt} = \frac{1}{2}m\omega^2 x_0^2 \ \omega \sin 2\omega t \Rightarrow P_{\max} = \frac{1}{2}m\omega^3 x_0^2 = \frac{1}{2}m\left(\frac{k}{m}\right)^{\frac{3}{2}} x_0^2 = \frac{1}{2}kx_0^2\sqrt{\frac{k}{m}}$$

Obviously, the Power is maximum when $2\omega t = \frac{\pi}{2} \Rightarrow \omega t = \frac{\pi}{4}$ So $x = \frac{x_0}{\sqrt{2}}$

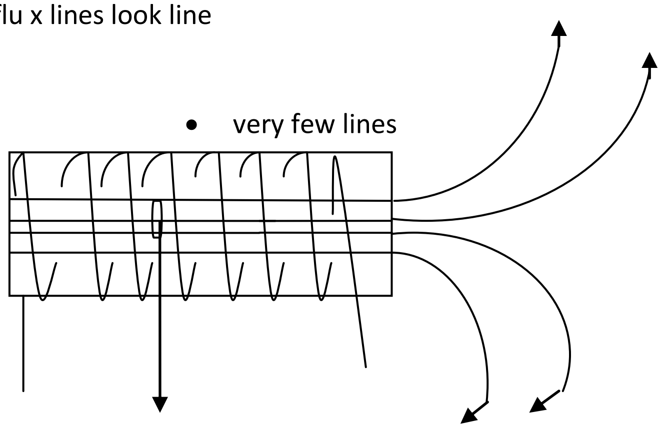
So $x = \frac{a}{\sqrt{2}}$ and $P_{\max} = \frac{1}{2}ka^2\sqrt{\frac{k}{m}}$

Ans: c

12. Total flux $= \frac{q}{\varepsilon_0}$ From the centre of a face the other five faces subtend $\frac{1}{2}$ of 4π solid angle so flux through the other five faces is $\frac{q}{2\varepsilon o}$

Ans: c

13. The magnetic flu x lines look line



maximum
Ans: a

14. $e = 312\,sin\left(100\pi t + \sqrt{2}\pi\right)$ The measured value of current isr.m.svalue $= \dfrac{E_0}{R\sqrt{2}} = \dfrac{312}{156} \cdot \dfrac{1}{\sqrt{2}} = \sqrt{2}$)

Ans: c

15. Conservation of momentum provides $Mv\hat{i} = \vec{p}_f + \dfrac{\hbar v}{c}\hat{j} \Rightarrow \vec{p}_f = -Mv\hat{i} + \dfrac{\hbar v}{c}\hat{j}$

$$\Rightarrow p_f^2 = \left(Mv\right)^2 + \left(\frac{\hbar v}{c}\right)^2$$

$Energy\ Conservation$ gives $\dfrac{1}{2}Mv^2 + \Delta E = \dfrac{p_f^2}{2M} + hv = \dfrac{\left(Mv\right)^2 + \left(\dfrac{hv}{c}\right)^2}{2M} + hv$ or $\Delta E = \left(\dfrac{hv}{c}\right)^2 \dfrac{1}{2M} + hv$

$\Rightarrow \dfrac{\left(hv\right)^2}{2Mc^2} + hv - \Delta E = 0$ Therefore $hv = \dfrac{-1 \pm \sqrt{1 + \dfrac{2\Delta E}{Mc^2}}}{\dfrac{1}{Mc^2}} = Mc^2\left(-1 \pm \sqrt{1 + \dfrac{2\Delta E}{Mc^2}}\right)$

$+\ Sign\ is\ the\ correct\ choice$ since the frequencycannot be negativetherefore

$as\ hv = Mc^2\left[\sqrt{1 + \dfrac{2\Delta e}{Mc^2}} - 1\right]$ OR $v = \dfrac{Mc^2}{h}\left[\sqrt{1 + \dfrac{2\Delta e}{Mc^2}} - 1\right]$

Ans: c

16. *Crab Nebula*.
Ans: b

17. 18 hours.
Ans: a

18. Southern latitude has longer days

Ans: a

19. For type II Luminosity would be $\frac{I}{4}$. Thus for the same apparent brightness distance would be less (nearer to the earth) by a factor 2. (Decreases by a factor 2).

Ans: d

20. If the number of Reactions $= N$ per sec. Then the energy released is $N \times 27 \times 1.6 \times 10^{-13}\,Js^{-1}$

Energy received on Earth $= \frac{N \times 27 \times 1.6 \times 10^{-13}}{4\pi R^2} = 1.4 \times 10^3 Wm^{-2}$

Each Reaction produces $2\ \nu_e$ Thus

Neutrino flux $\frac{2N}{4\pi R^2} = \frac{2 \times 1.4 \times 10^3}{27 \times 1.6 \times 10^{-13}} = 0.065 \times 10^{16} = 6.5 \times 10^{14} s^{-1} m^{-2}$

Ans: a

21. $\frac{4\pi}{3} r^3 N = \frac{4\pi}{3} R^3 \Rightarrow r = k \frac{R}{N^{\frac{1}{3}}}$

$\frac{1}{2} Mv^2 = \frac{GM^2 N^{\frac{1}{3}}}{2Rk} \Rightarrow v = \sqrt{\frac{GMN^{\frac{1}{3}}}{Rk}}$ where k =1

$v \propto \sqrt{\frac{GMN^{\frac{1}{3}}}{R}}$

Ans: a

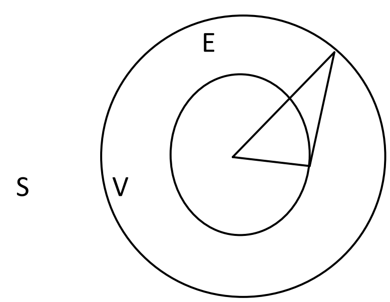
22. $SE = 150 \times 10^6 km$, $SV = 110 \times 10^6 km,\ EV = 90 \times 10^6 km$

Using cosine rule $SV^2 = SE^2 + EV^2 - 2\,SE\,.\,EV \cos \angle SEV$

$110^2 = 150^2 + 90^2 - 2 \times 150 \times 90 \cos \angle SEV$

$\cos \angle SEV = \frac{150^2 + 90^2 - 110^2}{2 \times 150 \times 90} = \frac{37}{54}$

$\Rightarrow \angle SEV = 46.7° \approx 47°$



Ans: b

23. $10°N$ Is the place where the star's altitude will be zero at its lowest point. So $10°N$ is answer

Ans: a

24. Moon moves $360°$ in approximately in 27.5 days. Therefore the angle moved is $\frac{360°}{27.5} = 13°$ eastward

Hours angle $= \left(40° - 13°\right) = 27°$ (Hour angle is measured Westward)

Ans: d

25. Foran ellipse the coefficients of $x^2 \& y^2$ $should\ be \rangle 0$ for hyperbole the coefficients of $x^2 \& y^2$ should have opposite sign

Ans: a & c   Answer d is wrong as if $b=0$ x+3=0 $x=-3(point)$

26. $\tan\theta = \cot\left(\frac{\pi}{3}\right) = \tan\frac{\pi}{6}$. tan is $+ve$ in the first and third quadrant.

Thus $\tan\frac{\pi}{6} = \tan\left(\frac{\pi}{6}+\pi\right) = \tan\left(\frac{\pi}{6}+2\pi\right) = \tan\left(\frac{\pi}{6}-\pi\right)$

$\theta = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$ $and\ not\ \frac{5\pi}{6}$ are possible

Ans: a,b,c.

27. $V_{critical} = \frac{\text{Re}ynold's\ number \times co.eff\ of\ vis\cos ity}{density \times diameter}$

Ans: a,b,c & d

28. The upper part of Lens can get light from all parts of the object. Hence Full Image will be on screen. Since only half the light passes, intensity will be reduced.

Ans: a, b & c .

29.The angular velocity of Point Q is $v = \omega r = \omega L \sin\theta \Rightarrow \vec{v} = \vec{\omega} \times \vec{L}$

The induced electric field is given by $\vec{E}_{induced} = \vec{v} \times \vec{B} = vB_0 \sin\theta = \omega r \sin\theta B_0 \sin\theta = r\omega B_0 \sin^2\theta$

Thus the electric field induced along the rod shall be $E = r\omega B_0 \sin^2\theta$

The induced emf in a rotating rod perpendicular to its length is

$d\varepsilon = -dlBv = -dr\sin\theta B_0 \omega r \sin\theta$ Thereby $\varepsilon = -B_0 \omega \sin^2\theta \int_0^L rdr$

or $\varepsilon = -\frac{1}{2} B_0 \omega L^2 \sin^2\theta$

Ans: a, b & d

30. The intensity and frequency of lines tells us about the amount and the kind of elements present.

Shift dueto Doppler Effect gives the radial velocity.

Ans: a & c .

31. R RLyare andEelipsing binary are the variable stars

Ans: b &d.

32. Globular clusters are in the outer regions of our Galaxy (Milky Way) and they contain old stars.

Ans: a