

**INTERNATIONAL OLYMPIAD QUALIFIER IN PHYSICS 2020-21**  
**Paper code 61 Solutions: (1.3.2021)**

1. Let time  $t$  depend on  $c$ ,  $h$  and  $G$  such that

$t = c^x h^y G^z$  Taking dimensions on both sides

$$M^0 L^0 T^1 = (L T^{-1})^x (M L^2 T^{-1})^y (M^{-1} L^3 T^{-2})^z$$

$$\text{or } M^0 L^0 T^1 = L^{x+2y+3z} M^{y-z} T^{-x-y-2z}$$

$$\text{Giving } y-z=0 \quad (1) \quad x+2y+3z=0 \quad (2) \quad -x-y-2z=0 \quad (3)$$

Or  $y=z$  Putting in (2) we get  $x=-5y$  then from (3)  $5y-y-2y=1$

$$\Rightarrow 2y=1 \Rightarrow y=\frac{1}{2}, \quad x=-\frac{5}{2} \text{ and } z=\frac{1}{2} \text{ So we get } t=c^{-\frac{5}{2}} h^{\frac{1}{2}} G^{\frac{1}{2}} \Rightarrow \sqrt{\frac{hG}{c^5}}$$

Ans: d

2. The mass of the composite system is  $M = \frac{2}{3} \pi R^3 \rho + \pi R^2 \ell \rho = \frac{2}{3} \pi R^3 \rho \left(1 + \frac{3\ell}{2R}\right)$

The moment of Inertia is  $I = \frac{2}{5} \times \frac{2}{3} \pi R^3 \rho R^2 + \frac{1}{2} \pi R^2 \ell \rho R^2 = \frac{4}{15} \pi R^5 \rho \left(1 + \frac{15\ell}{8R}\right)$  using now

$$I = MK^2 \text{ we get } K = \frac{\sqrt{\frac{4}{15} \pi R^5 \rho \left(1 + \frac{15\ell}{8R}\right)}}{\frac{2}{3} \pi R^3 \rho \left(1 + \frac{3\ell}{2R}\right)} = R \sqrt{\frac{1}{10} \frac{(8R+15\ell)}{(2R+3\ell)}}$$

Ans: b

3.  $g' = g - \omega^2 R$  where  $g'$  is apparent acceleration due to gravity and  $\omega = \frac{2\pi}{T}$  is the

angular velocity at the verge of fly off means  $g' = 0$  or  $g = \omega^2 R = \left(\frac{2\pi}{T}\right)^2 R$  Thereby

$$\frac{G}{R^2} \frac{4}{3} \pi R^3 \rho = \left(\frac{2\pi}{T}\right)^2 R \Rightarrow T = \sqrt{\frac{3\pi}{\rho G}}$$

Ans: c

4. Let the stationary mass  $m$  explodes in to  $m_1$  and  $m_2$

By conservation of momentum  $m_1 v_1 + m_2 v_2 = 0$  (1) and Energy is

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = 1680 \Rightarrow \frac{1}{2} m_1 v_1^2 \left(1 + \frac{m_1}{m_2}\right) = 1680$$

$$\Rightarrow v_1 = 12 \text{ ms}^{-1} \text{ and } \Rightarrow v_2 = -28 \text{ ms}^{-1} \text{ Thereby } v_1 - v_2 = [12 - (-28)] \text{ ms}^{-1} = 40 \text{ ms}^{-1}$$

Thus we get  $v_{rel} = 40 \text{ ms}^{-1}$

Ans: a

5. For a projectile the maximum height is  $H = \frac{u^2 \sin^2 \alpha}{2g}$  and the range is  $R = \frac{u^2 \sin 2\alpha}{g}$

For the given problem  $\frac{H}{R/2} = \tan \beta \Rightarrow \frac{\sin^2 \alpha}{\sin 2\alpha} = \tan \beta \Rightarrow \tan \alpha = 2 \tan \beta$

Ans: a

6. As particle starts from rest, it must have started from extreme position. So equation of SHM is  $x = A \cos \omega t$ , where A is amplitude and  $x$  displacement from Centre.

Given that at  $t = 1$ ,  $x = A - a \Rightarrow A - a = A \cos(\omega \times 1)$ .....(1) and

at  $t = 2$ ,  $x = A - a - b \Rightarrow A - a - b = A \cos(\omega \times 2)$ .....(2)

Using  $\cos 2\omega = 2 \cos^2 \omega - 1$  one obtains  $A = \frac{2a^2}{3a - b}$

Ans: a

7. The velocity is changing on circular path so centripetal acceleration and tangential acceleration

both Further Given is that  $\frac{1}{2}mv^2 = as^2 \Rightarrow \frac{mv^2}{R} = \frac{2as^2}{R}$  centripetal force

Also  $mv^2 = 2as^2 \Rightarrow v = \sqrt{\frac{2a}{m}} s \Rightarrow \frac{dv}{dt} = \sqrt{\frac{2a}{m}} \frac{ds}{dt} = \sqrt{\frac{2a}{m}} \sqrt{\frac{2a}{m}} s$

$\Rightarrow \frac{dv}{dt} = \frac{2a}{m} s \Rightarrow m \frac{dv}{dt} = 2as =$  tangential force

Net force as a function of  $s$  is  $F = \sqrt{F_T^2 + F_R^2} \Rightarrow 2as \sqrt{1 + \left(\frac{s}{R}\right)^2}$

Ans: d

8. From eq. of continuity  $A_1 v_1 = A_2 v_2$ . -----(1)

Given that  $A_1 = 10 \text{ cm}^2$  and  $A_2 = 5 \text{ cm}^2$  and  $v_1 = 1 \text{ m/s}$  Putting in (1) gives  $v_2 = 2 \text{ m/s}$

For horizontal tube from Bernoulli eq.

$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$ .-----(2)

Now put in eq (2)  $P_1 = 2000 \text{ Pa}$ ,  $\rho = 10^3 \text{ kg/m}^3$ . Also  $v_1 = 1 \text{ m/s}$  and  $v_2 = 2 \text{ m/s}$

And get  $P_2 = 500 \text{ Pa}$ .

Ans: d

9. Let  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  be the temperature of water in the three containers.

When one litre from A and two litre from B is mixed, we get

$(\theta_1 - 52) = 2(52 - \theta_2) \Rightarrow \theta_1 + 2\theta_2 = 3 \times 52$

When one litre from B and two litre from C is mixed, we get

$(\theta_2 - 40) = 2(40 - \theta_3) \Rightarrow \theta_2 + 2\theta_3 = 3 \times 40$

When one litre from C and two litre from A is mixed, we get

$$(\theta_3 - 34) = 2(34 - \theta_1) \Rightarrow \theta_3 + 2\theta_1 = 3 \times 34$$

Adding the three, we get  $3(\theta_1 + \theta_2 + \theta_3) = 3 \times (52 + 40 + 34) \Rightarrow \theta_1 + \theta_2 + \theta_3 = 126$  If  $\theta_0$  is the temperature when one litre from each A, B and C is mixed then

$$(\theta_1 - \theta_0) = (\theta_0 - \theta_2) + (\theta_0 - \theta_3) \Rightarrow \theta_1 + \theta_2 + \theta_3 = 3\theta_0 \Rightarrow \theta_0 = 42^\circ$$

Ans: b

10. The force along x axis is

$$F_x = \frac{1}{4\pi\epsilon_0} \left[ \frac{q^2}{\ell^2} + 2 \frac{q^2}{(\ell\sqrt{2})^2} \frac{1}{\sqrt{2}} + \frac{q^2}{(\ell\sqrt{3})^2} \frac{1}{\sqrt{3}} \right] = \frac{1}{4\pi\epsilon_0} \frac{q^2}{\ell^2} \left[ 1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}} \right] \text{ Similar}$$

expressions are held along y and z axes. Hence the resultant force is

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \Rightarrow$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{\ell^2} \left[ 1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}} \right] \sqrt{\{1+1+1\}} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{\ell^2} \left[ \sqrt{3} + \sqrt{\frac{3}{2}} + \frac{1}{3} \right] = \frac{0.8225 q^2}{\pi\epsilon_0 \ell^2}$$

$$\text{Or } F = \frac{(1-0.1775) q^2}{\pi\epsilon_0 \ell^2}$$

Ans: c

11. The internal resistance 'r' of a cell is measured by a potentiometer as

$r = \left(\frac{L}{L_1} - 1\right) R_1 = \left(\frac{L}{L_2} - 1\right) R_2$  where L is the balancing length when cell is in open circuit and  $L_1$  when the cell is shunted by resistance  $R_1$  and  $L_2$  when cell is shunted by resistance  $R_2$ . Given that  $L=250\text{cm}$ ,  $R_1=7.5\ \Omega$ ,  $L_1=250-25=225\text{cm}$ ,  $R_2=20\ \Omega$ , This gives  $L_2=240\text{cm}$

Ans: a

12. Knowing that  $C_V = \frac{R}{(\gamma-1)}$ ,  $C_P = \frac{\gamma R}{(\gamma-1)}$  and  $\gamma_{mix} = \frac{n_1 C_{P1} + n_2 C_{P2}}{n_1 C_{V1} + n_2 C_{V2}}$  with

$$\gamma_1 = \frac{5}{3} \text{ and } \gamma_2 = \frac{7}{5} \text{ and } n_1 = 1, \text{ \& } n_2 = 2, \text{ we get}$$

$$C_{V1} = \frac{3}{2}R, C_{P1} = \frac{5}{2}R \text{ and } C_{V2} = \frac{5}{2}R, C_{P2} = \frac{7}{2}R \text{ and obtain}$$

$$\gamma_{mix} = \frac{n_1 C_{P1} + n_2 C_{P2}}{n_1 C_{V1} + n_2 C_{V2}} = \frac{19}{13} = 1.46$$

Ans: b

13. Given that  $PT^3 = K$ , using  $PV = \mu RT$  or  $P = \frac{\mu RT}{V}$  we get

$$\frac{\mu RT}{V} T^3 = K \text{ or } T^4 = \frac{KV}{\mu R} \quad \text{differentiating we get } 4T^3 dT = \frac{KdV}{\mu R} \text{ Thereby}$$

$$\frac{dV}{dT} = \frac{4\mu RT.T^2}{K} = \frac{4V}{T} \text{ so coefficient of volume expansion } \frac{1}{V} \frac{dV}{dT} = \frac{4}{T}$$

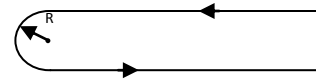
Ans: d

14. Magnetic field at the centre of arc O is due to semi-circular part and to two semi infinite straight lines.

$$\text{So } B = \frac{\mu_0 I}{4R} + 2 \left[ \frac{\mu_0 I}{4\pi R} (\sin 0 + \sin 90) \right] = \frac{\mu_0 I}{4R} \left( 1 + \frac{2}{\pi} \right)$$

$$B = \frac{\mu_0 I}{4\pi R} (\pi + 2)$$

Ans: a



15. By symmetry net electric field along X-axis at the centre O is zero and the electric field along y axis will be added up

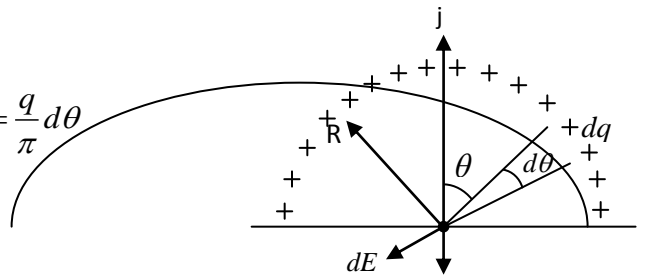
$$dE_y = (-\hat{j}) \frac{1}{4\pi\epsilon_0} \frac{dq}{R^2} \cos \theta$$

$$\text{where } dq = \lambda(Rd\theta) = \frac{q}{\pi R} (Rd\theta) = \frac{q}{\pi} d\theta$$

$$E_y = (-\hat{j}) 2 \times \frac{1}{4\pi\epsilon_0} \int_0^{\pi/2} \left( \frac{q}{\pi} \cos \theta d\theta \right) \frac{1}{R^2}$$

$$E_y = (-\hat{j}) \frac{q}{2\pi^2 \epsilon_0 R^2}$$

Ans: a



16. The current is  $i = i_1 \cos \omega t + i_2 \sin \omega t = i_1 \left( \cos \omega t + \frac{i_2}{i_1} \sin \omega t \right)$  let us put

$$\frac{i_2}{i_1} = \cot \theta = \frac{\cos \theta}{\sin \theta} \text{ then } i = \frac{i_1}{\sin \theta} (\cos \omega t \sin \theta + \sin \omega t \cos \theta) \text{ or}$$

$$i = \frac{i_1}{\sin \theta} \sin(\omega t + \theta) \text{ or } i = \sqrt{(i_1^2 + i_2^2)} \sin(\omega t + \theta) \text{ Thereby rms current is}$$

$$i_{rms} = \sqrt{\frac{(i_1^2 + i_2^2)}{2}}$$

Ans: c

17. The potential at the origin may be expressed as

$$V_0 = \frac{1}{4\pi\epsilon_0} \left( \frac{q}{x_0} - \frac{q}{2x_0} + \frac{q}{3x_0} - \frac{q}{4x_0} + \frac{q}{5x_0} \dots \right)$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{x_0} \left( 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots \right) \text{ using now}$$

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} \dots \text{ for } -1 < x \leq 1$$

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \dots$$

$$\text{One obtains } V = \frac{1}{4\pi\epsilon_0} \frac{q}{x_0} \ln 2$$

Ans: b

18. The mass defect  $\Delta m = (\text{mass of } {}^{23}_{10}\text{Ne} - \text{mass of 10 electrons}) - (\text{mass of } {}^{23}_{11}\text{Na} - \text{mass of 11 electrons}) - \text{mass of one electron}$

$$\text{Or } \Delta m = (\text{mass of } {}^{23}_{10}\text{Ne}) - (\text{mass of } {}^{23}_{11}\text{Na}) = (22.994466) - (22.989770) = 0.004696 \text{ amu}$$

Thereby  $\Delta E = 0.004696 \times 931.5 = 4.374 \text{ MeV}$  This energy is shared between the electron and the neutrino. In an extreme situation the electron can take the whole of this energy so the maximum energy the electron can have is 4.374 MeV

Ans: a

19. The resultant intensity on the screen is given by  $I = I_m \cos^2 \left( \frac{\pi}{\lambda} d \frac{y}{D} \right) = \frac{1}{4} I_m$

$$\text{Thereby } \cos^2 \left( \frac{\pi}{\lambda} d \frac{y}{D} \right) = \frac{1}{4} \Rightarrow \frac{\pi}{\lambda} d \frac{y}{D} = \frac{\pi}{3} \Rightarrow y = \frac{D\lambda}{3d}$$

$$y = \frac{1.20 \times 600 \times 10^{-9}}{3 \times 2.5 \times 10^{-3}} = 96 \times 10^{-6} \text{ m} = 96 \mu\text{m}$$

Ans: c

20. Let the initial current through the coil be  $I_0$  at  $t=0$ . The current decrease down to zero halving after each  $\Delta t$  second This means that the current at any time  $t$  is expressed as

$$i = I_0 e^{-\frac{\ln 2}{\Delta t} t}. \text{ The heat produced in time } dt \text{ in the coil is } dH = i^2 R dt. \text{ The total heat}$$

produced will be  $H = \int_0^{\infty} i^2 R dt$  Substituting the values

$$H = \int_0^{\infty} I_0^2 e^{-2\frac{\ln 2}{\Delta t} t} R dt = I_0^2 R \int_0^{\infty} e^{-2\frac{\ln 2}{\Delta t} t} dt = \frac{I_0^2 R \Delta t}{-2 \ln 2} \left[ e^{-2\frac{\ln 2}{\Delta t} t} \right]_0^{\infty} = \frac{I_0^2 R \Delta t}{2 \ln 2} \dots (1) \text{ Also we know}$$

$$i = \frac{dQ}{dt} \quad \text{or} \quad dQ = idt \quad \text{or} \quad Q = \int_0^q dQ = \int_0^{\infty} idt = \int_0^{\infty} I_0 e^{-\frac{\ln 2}{\Delta t} t} dt = \left\{ -\frac{I_0 \Delta t}{\ln 2} e^{-\frac{\ln 2}{\Delta t} t} \right\}_0^{\infty} = \frac{I_0 \Delta t}{\ln 2} \quad \text{or}$$

$$I_0 = \frac{Q \ln 2}{\Delta t} \quad \text{Substituting in (1)} \quad H = \frac{\left( \frac{Q \ln 2}{\Delta t} \right)^2 R \Delta t}{2 \ln 2} = \frac{1}{2} \frac{Q^2 R}{\Delta t} \ln 2 \quad \text{Ans}$$

Ans: c

21. When switch S is closed, current starts flowing and is given by

$$I = I_0 (1 - e^{-\frac{t}{\tau}}) = \frac{E}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) = \frac{dq}{dt} \quad \text{Therefore} \quad q = \int_0^{\tau} I dt = q = \frac{E}{R} \int_0^{\tau} \left( 1 - e^{-\frac{t}{\tau}} \right) dt$$

Where charge  $q$  flows in time  $\tau$  ( $\because \tau = \frac{L}{R}$  = time constant)

$$q = \frac{E}{R} \tau - \frac{E}{R} \int_0^{\tau} e^{-\frac{t}{\tau}} dt = \frac{E}{R} \tau - \frac{E}{R} \left[ \frac{e^{-\frac{t}{\tau}}}{\left( \frac{-1}{\tau} \right)} \right]_0^{\tau} = \frac{E}{R} \tau + \frac{E}{R} \tau \left[ e^{-\frac{t}{\tau}} \right]_0^{\tau}$$

$$q = \frac{E}{R} \tau + \left( \frac{E}{R} \tau e^{-1} - \frac{E}{R} \tau e^0 \right) = \frac{E\tau}{eR} = \frac{E \left( \frac{L}{R} \right)}{eR} = \frac{EL}{eR^2}$$

Ans: a

22. In a sample of uranium of mass  $M$ , the masses of the two isotopes are

$$M_1 = \frac{140}{141} M \quad \text{and} \quad M_2 = \frac{1}{141} M \quad \text{The number of atoms of the two isotopes are}$$

$$N_1 = \frac{140}{141} M \frac{N_A}{238} \quad \text{and} \quad N_2 = \frac{1}{141} M \frac{N_A}{235} \quad \text{Knowing further that } N = N_0 e^{-\lambda t} \text{ gives the Activity}$$

$$\text{as } A_1 = -\frac{dN_1}{dt} = \lambda N_1 = \frac{\ln 2}{T_1} \frac{140}{141} M \frac{N_A}{238} \quad \text{and} \quad A_2 = -\frac{dN_2}{dt} = \lambda N_2 = \frac{\ln 2}{T_2} \frac{1}{141} M \frac{N_A}{235} \quad \text{The}$$

relative contribution of Activity thus turns out to be  $\frac{A_1}{A_1 + A_2} : \frac{A_2}{A_1 + A_2} \Rightarrow$

$$\frac{1}{4.5} \times \frac{140}{238} : \frac{1}{0.7} \times \frac{1}{235}$$

$$\Rightarrow 0.1307 : 0.0060 :: \frac{0.1307}{0.1367} \times 100 : \frac{0.0061}{0.1367} \times 100 \Rightarrow 95.6\% \quad \text{and} \quad 4.4\%$$

Ans: c

23. The focal length of a lens is obtained by  $\frac{\mu_2}{f_2} = \frac{\mu - \mu_1}{R_1} - \frac{\mu - \mu_2}{R_2}$  .....(1) where  $\mu, \mu_1$  &  $\mu_2$

are the refractive indices of the material of lens, the object space and the image space respectively.  $R_1$  &  $R_2$  are the two radii of the lens.  $f_2$  is the second focal length. When the lens is

placed in air  $\mu = 1.5, \mu_1 = 1$  &  $\mu_2 = 1$  and then  $\frac{1}{25} = \frac{1}{f} = \frac{1.5-1}{R} - \frac{1.5-1}{-R} \Rightarrow R = 25$

In the present case  $\mu = 1.5, \mu_1 = 1$  &  $\mu_2 = \frac{4}{3}$  Then equation (1) yields  $\frac{4}{3f_2} = \frac{\frac{3}{2}-1}{25} - \frac{\frac{3}{2}-\frac{4}{3}}{-25}$

$\Rightarrow f_2 = 50$  cm Hence the sun will be focused 50 below the lens.

Ans: c

24. Lyman series of hydrogen spectrum falls in ultra violet region. Minimum energy photon of Lyman series is emitted for transition from  $n=2$  to  $n=1$  and has an energy  $13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$  eV = 10.2 eV. All other spectral lines will be of higher energies and so the frequencies. Therefore if this 10.2 eV photon can eject photo electron then other will definitely. So required threshold frequency  $\nu$  is

$$\nu = \frac{10.2 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} \text{ Hz} = 2.46 \times 10^{15} \text{ Hz}$$

Ans : c

### Multiple choice questions (Any number of options may be correct)

25. When battery has been disconnected, the charge  $Q$  remains unchanged

$$Q = C_{air} V = \left(\frac{\epsilon_0 A}{d}\right) V = k \epsilon_0 A E$$

Electric field in dielectric between plates of capacitor  $E = \frac{\sigma}{K \epsilon_0} = \frac{E_{air}}{K} = \frac{V}{Kd}$

$$\text{Work done on the system} = \frac{Q^2}{2C_{air}} - \frac{Q^2}{2C_{dielectric}} = \frac{Q^2}{2} \left[ \frac{1}{C_{air}} - \frac{1}{C_{dielectric}} \right]$$

$$W = \frac{1}{2} \left( \frac{\epsilon_0 A V}{d} \right)^2 \left( \frac{d}{\epsilon_0 A} - \frac{d}{k \epsilon_0 A} \right) = \frac{\epsilon_0 A V^2}{2d} \left[ 1 - \frac{1}{k} \right]$$

Ans: a, & c

26. The gravitational field due to a uniform solid sphere of mass  $M$  and radius  $R$  at a distance  $r$  from its centre is

$$F(r) = \left(\frac{GM}{R^3}\right) r \text{ if } r < R \text{ and } F(r) = \frac{GM}{r^2} \text{ if } r > R \text{ Thereby}$$

$$\frac{F(r_1)}{F(r_2)} = \frac{r_1}{r_2} \text{ for } r_1 \leq R \text{ and } r_2 \leq R \text{ and } \frac{F(r_1)}{F(r_2)} = \frac{r_2^2}{r_1^2} \text{ for } r_1 \geq R \text{ and } r_2 \geq R$$

Ans: a & b

27. The resultant intensity is  $I_0 = I + I + 2\sqrt{I \times I} \cos 0 = 4I$  when the intensity of one source is reduced by 64 % it becomes  $I - 0.64I = 0.36I$  Then the resultant intensity becomes

$$I_{\text{Result}} = 0.36I + I + 2\sqrt{0.36I \times I} \cos \phi = I_0 (0.34 + 0.30 \cos \phi) \text{ where phase } \phi \text{ is now varied.}$$

When  $\phi = 0$ , The intensity at P is  $I_{\text{Result}} = 0.64I_0 = I_{\text{max}}$  Ans a

When  $\phi = \frac{\pi}{2}$ . The intensity at P is  $I_{\text{Result}} = 0.34I_0$

When  $\phi = \pi$  The intensity at P is  $I_{\text{Result}} = 0.04I_0 = I_{\text{min}}$

This shows that  $\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{0.64}{0.04} = 16$  Ans c

Ans: a & c

28. The given parameters are  $P_i = 1 \times 10^5 \text{ N/m}^2, V_i = 2 \times 10^{-3} \text{ m}^3$ ,

$$P_f = P_o + \frac{kx}{A} = 1.5 \times 10^5 \text{ N/m}^2 \text{ and } V_f = V_o + Ax = 2.4 \times 10^{-3} \text{ m}^3 \text{ Now } T_f = \frac{P_f \times V_f}{P_i V_i} T_i = 720 \text{ K}$$

$$\text{And } \Delta U = nC_v dT = \frac{nR}{\gamma - 1} dT = \frac{P_i V_i}{\gamma - 1} \frac{dT}{T} = 240 \text{ J } \Delta W = P_o Ax + \frac{1}{2} kx^2 = 50 \text{ J} \text{ Then}$$

$$\Delta Q = \Delta U + \Delta W = 290 \text{ J}$$

Ans: a, b & d

29. Given that  $U_x = U_0(1 - \cos ax)$  The force  $F = -\frac{dU_x}{dx} = -aU_0 \sin ax$  For small displacement x it

turns out to  $F = -\frac{dU_x}{dx} = -a^2 U_0 x$  Obviously the force is zero at  $x = 0$  showing that  $x = 0$  is the equilibrium position. The equilibrium is stable as the second derivative of potential function is negative. Once again  $m \frac{d^2 x}{dt^2} = -a^2 U_0 x$  is the equation of SHM whose time period is  $T = 2\pi \sqrt{\frac{m}{a^2 U_0}}$

and its angular frequency is  $\omega = \sqrt{\frac{a^2 U_0}{m}} = a \sqrt{\frac{U_0}{m}}$

Ans: a, b, c & d

30. The refractive index of the prism is  $\mu = 1.6 = \frac{\sin\left(\frac{A + \hat{\partial}_m}{2}\right)}{\sin \frac{A}{2}}$  where  $\hat{\partial}_m$  is the angle of

minimum deviation. Using  $\mu = 1.6$  and  $A = 60^\circ$  One gets,

$\sin \frac{A + \partial_m}{2} = 0.8 \Rightarrow \frac{A + \partial_m}{2} = 53^\circ \Rightarrow \partial_m = 46^\circ$  **Ans b** Also the angle of incidence here is

$$i = \left( \frac{A + \partial_m}{2} \right) = \frac{60 + 46}{2} = 53^\circ \text{ Ans a}$$

Now the prism is immersed in water of refractive index  ${}_a\mu_w = \frac{4}{3}$ , the angle of minimum deviation may

$$\text{now be obtained from } {}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.6}{4/3} = 1.2 = \frac{\sin\left(\frac{A + \partial_m}{2}\right)}{\sin\frac{A}{2}} \Rightarrow \left(\frac{A + \partial_m}{2}\right) = 37^\circ \Rightarrow \partial_m = 14^\circ \text{ A}$$

When immersed in a liquid of refractive index  ${}_a\mu_l = 1.2$ , The deviation may be obtained from

$${}_l\mu_g \sin \frac{A}{2} = \sin\left(\frac{A + \partial_m}{2}\right) \text{ or } \frac{1.6}{1.2} \sin \frac{60}{2} = \sin\left(\frac{60 + \partial_m}{2}\right) \Rightarrow \partial_m = 23.6^\circ$$

Ans: a, b, c & d

31. The current in a p – n junction diode is expressed as

$$i = i_0 \left( e^{qV/kT} - 1 \right) \text{ At } 300 \text{ K, the value of } \frac{qV}{kT} = \frac{1.6 \times 10^{-19} \times 0.6}{1.38 \times 10^{-23} \times 300} = 23.2 \text{ Therefore the current}$$

$$i = 5 \times 10^{-12} \left( e^{23.2} - 1 \right) = 5.0 \times 10^{-12} \times 1.190 \times 10^{10} \text{ or } i = 0.0595 \text{ A} \cong 59.5 \text{ mA} \text{ Now when}$$

$$V = 0.7 \text{ V The value of } \frac{qV}{kT} = \frac{1.6 \times 10^{-19} \times 0.7}{1.38 \times 10^{-23} \times 300} = 27.053 \text{ and the current } i = i_0 \left( e^{27.053} - 1 \right)$$

$$i = 5.0 \times 10^{-12} \times 5.610 \times 10^{11} \Rightarrow i = 2.805 \text{ A}$$

Thereby the change in current when voltage is changed from 0.6 V to 0.7 V is

$$\Delta i = 2.805 - 0.0595 \text{ A} \cong 2.75 \text{ A}$$

For dynamic resistance, using now  $i = i_0 \left( e^{qV/kT} - 1 \right)$  we get  $\left. \frac{di}{dV} \right|_T = \frac{qi_0}{kT} \left( e^{qV/kT} \right)$ .

$$\text{At } V = 0.6 \text{ volt, } \left. \frac{di}{dV} \right|_{T=300K} = \frac{qi_0}{kT} \left( e^{23.2} \right) = \frac{1.6 \times 5}{1.38 \times 3} 10^{-10} \times 1.19 \times 10^{10} \cong 2.37$$

Therefore the dynamic resistance of the diode at a biasing voltage of 0.6 volt is

$$R_d = \left. \frac{dV}{di} \right|_{V=0.6V, T=300K} = \frac{1}{2.3} = 0.435 = 435 \text{ m}\Omega$$

In the reverse bias the current practically remains constant up to a large value known as break down voltage so no change in reverse bias current occurs when voltage changes from -1V to -2V

Ans: a, b, c & d

average

32. In the absence of electric field, when the drop falls under gravity alone, its

$$\text{speed is } v = \frac{\text{distance covered}}{\text{time}} \Rightarrow v = \frac{2.0 \times 10^{-3} \text{ m}}{35.7 \text{ s}} = 0.056 \frac{\text{mm}}{\text{s}} \quad \text{Also the next}$$

$$v = \frac{1.2 \times 10^{-3} \text{ m}}{21.4 \text{ s}} = 0.056 \frac{\text{mm}}{\text{s}} \quad \text{This shows that this is the terminal velocity.}$$

Therefore the apparent weight of the drop of radius  $r$  and density  $\rho$  equals the viscous force, that

$$\text{is, } \frac{4}{3} \pi r^3 (\rho - \sigma) g = 6\pi\eta r v \quad \text{Where } \sigma \text{ is the density and } \eta \text{ the viscosity of air.}$$

$$\text{Thus, } r = \sqrt{\frac{9\eta v}{2(\rho - \sigma)g}} \quad \text{or } r = \sqrt{\frac{9 \times (1.80 \times 10^{-5}) \times (0.056 \times 10^{-3})}{2 \times (880 - 1.29) \times 9.81}} \Rightarrow r = 7.26 \times 10^{-7} \text{ m}$$

When the drop is held stationary in the electric field, the upward electric force on the drop equals the apparent weight of the drop. That is,

$$qE = \frac{4}{3} \pi r^3 (\rho - \sigma) g \quad \text{or } q = \frac{4\pi r^3 (\rho - \sigma) g}{3E} \quad \text{Here } E = \frac{V}{d} = \frac{103}{6.0 \times 10^{-3}} \text{ Vm}^{-1}$$

$$\therefore q = \frac{4 \times 3.14 \times (7.26 \times 10^{-7})^3 \times (880 - 1.29) \times 9.81}{3 \times 103 / 6.0 \times 10^{-3}}$$

$$\text{or } q = \frac{4 \times 3.14 \times (7.26)^3 \times 878.71 \times 9.81 \times 6.0}{3 \times 103} \times 10^{-24} = 8.045 \times 10^{-19} \text{ C} \quad \text{to achieve equilibrium this}$$

charge must be negative

Now  $q = ne$ , where  $n$  is the number of excess electrons on the drop. Therefore,

$$n = \frac{q}{e} = \frac{8.045 \times 10^{-19}}{1.6 \times 10^{-19}} = 5 \quad \text{Thus the drop carries 5 excess electrons.}$$

Ans: a, c & d