



INDIAN ASSOCIATION OF PHYSICS TEACHERS

National Standard Examination in Astronomy - 2023

Date of Examination: December 10, 2023

Time: 9:00 AM to 11:00 AM

Question Paper Code: 45

Student's Roll No:									
-----------------------	--	--	--	--	--	--	--	--	--

Write the Question Paper code (mentioned above) on YOUR OMR Answer Sheet (in the space provided), otherwise your Answer Sheet will NOT be evaluated. Note that the same Question Paper Code appears on each page of the Question Paper.

Instructions to Candidates:

1. Use of mobile phone, smart watch, and iPad during examination is **STRICTLY PROHIBITED**.
2. In addition to this Question Paper, you are given OMR Answer Sheet along with candidate's copy.
3. On the Answer Sheet, make all the entries carefully in the space provided **ONLY** in **BLOCK CAPITALS** as well as by properly darkening the appropriate bubbles. **Incomplete/ incorrect/ carelessly filled information may disqualify your candidature.**
4. On the OMR Answer Sheet, use only **BLUE or BLACK BALL POINT PEN** for making entries and filling the bubbles.
5. Your **Ten-digit roll number and date of birth** entered on the OMR Answer Sheet shall remain your login credentials (means login id and password respectively) for accessing your performance / result in National Standard Examination in Astronomy – 2023.
6. Question paper has two parts. In part A1 (Q. No.1 to 48) each question has four alternatives, out of which **only one** is correct. Choose the correct alternative and fill the appropriate bubble, as shown.

Q.No.22

<input type="radio"/> a	<input checked="" type="radio"/>	<input type="radio"/> c	<input type="radio"/> d
-------------------------	----------------------------------	-------------------------	-------------------------

In part A2 (Q. No. 49 to 60) each question has four alternatives out of which any number of alternative (s) (1, 2, 3, or 4) may be correct. You have to choose **all** correct alternative(s) and fill the appropriate bubble(s), as shown

Q.No.54

<input type="radio"/> a	<input checked="" type="radio"/>	<input type="radio"/> c	<input checked="" type="radio"/>
-------------------------	----------------------------------	-------------------------	----------------------------------

7. For **Part A1**, each correct answer carries 3 marks whereas 1 mark will be deducted for each wrong answer. In **Part A2**, you get 6 marks if all the correct alternatives are marked. No negative marks in this part.
8. Rough work may be done in the space provided. There are **11** printed pages in this paper
9. Use of **Non - programmable scientific** calculator is allowed.
10. No candidate should leave the examination hall before the completion of the examination.
11. After submitting Answer Paper, take away the Question Paper & candidate's copy of OMR sheet for your future reference.

Please DO NOT make any mark other than filling the appropriate bubbles properly in the space provided on the OMR Answer Sheet.

Answer Sheets are evaluated using machine, hence CHANGE OF ENTRY IS NOT ALLOWED. Scratching or overwriting may result in wrong score.

DO NOT WRITE ON THE BACK SIDE OF THE ANSWER SHEET.

Instructions to Candidates (Continued) :

You may read the following instructions after submitting the Answer Sheet.

12. Comments/Inquiries/Grievances regarding this Question Paper, if any, can be shared on the Inquiry/Grievance column on www.iapt.org.in on the specified format till December 15, 2023

13. The answers/solutions to this Question Paper will be available on the website: www.iapt.org.in by December 14, 2023.

14. CERTIFICATES and AWARDS:

Following certificates shall be awarded by IAPT to the students, successful in the NATIONAL STANDARD EXAMINATION IN ASTRONOMY – 2023

- (i) “CENTRE TOP 10 %” To be downloaded from iapt.org.in after 30.01.24
- (ii) “STATE TOP 1 %” Will be dispatched to the examinee
- (iii) “NATIONAL TOP 1 %” Will be dispatched to the examinee
- (iv) “GOLD MEDAL & MERIT CERTIFICATE” to all students who attend OCSC – 2024 at HBCSE Mumbai

Certificate for centre toppers shall be uploaded on iapt.org.in

15. List of students (with centre number and roll number only) having score above **Minimum Admissible Score (MAS)** will be displayed on the website: www.iapt.org.in by **December 25, 2023**. See the MAS clause on the Student’s brochure on the web.

16. List of students eligible to appear for Indian National Astronomy Olympiad (INAO – 2024) shall be displayed on www.iapt.org.in by December 30, 2023.

Physical and Astronomical constants you may need....

Magnitude of charge on electron $e = 1.60 \times 10^{-19} C$	Speed of light in free space $c = 3 \times 10^8 \text{ ms}^{-1}$
Mass of electron $m_e = 9.11 \times 10^{-31} \text{ kg}$	Speed of sound in dry air at 0°C $v = 332 \text{ ms}^{-1}$
Mass of proton $m_p = 1.67 \times 10^{-27} \text{ kg}$	Permittivity of free space $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2$
Acceleration due to gravity $g = 9.81 \text{ ms}^{-2}$	Permeability of free space $\mu_0 = 4\pi \times 10^{-7} \text{ H / m}$
Universal gravitational constant $G = 6.67 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2$	Planck’s constant $h = 6.626 \times 10^{-34} \text{ Js}$
Universal gas constant $R = 8.31 \text{ J / mol K}$	Rydberg constant $R = 1.097 \times 10^7 \text{ m}^{-1}$
Boltzmann constant $k = 1.38 \times 10^{-23} \text{ J / K}$	Astronomical unit $1 \text{ AU} = 1.50 \times 10^{11} \text{ m}$
Stefan’s constant $\sigma = 5.67 \times 10^{-8} \text{ W / m}^2 \text{ K}^4$	Radius of Sun = $6.96 \times 10^8 \text{ m}$
Avogadro’s constant $A = 6.022 \times 10^{23} \text{ mol}^{-1}$	Mass of Sun $M = 2.0 \times 10^{30} \text{ kg}$
Faraday constant = $96,500 \text{ C / mol}$	Mass of Earth = $5.972 \times 10^{24} \text{ kg}$
1 parsec = $3.09 \times 10^{16} \text{ m}$	Radius of Earth = 6400 km

**INDIAN ASSOCIATION OF PHYSICS TEACHERS
NATIONAL STANDARD EXAMINATION IN ASTRONOMY
(NSEA - 2023)**

Time: 120 minute

Max. Marks: 216

Attempt All Sixty Questions

A – 1

ONLY ONE OUT OF FOUR OPTIONS IS CORRECT. BUBBLE THE CORRECT OPTION.

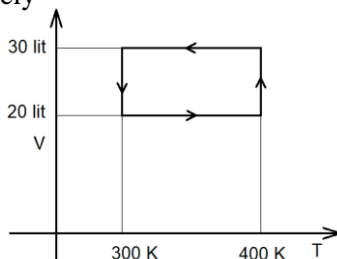
1. Venus has an equatorial radius of 6052 km. The semi-major axis of its orbit is 0.72 AU. The Sun has a radius of 7×10^{10} cm. A Venus transit is observed from Earth. The angular diameter of Venus, in arc seconds, when observed during the transit is
 (a) 60 (b) 30 (c) 0.6 (d) 1.2
2. A telescope with a 5 meter objective mirror diameter gathers times more light and can resolve objects of times angular size when compared to a 1 meter telescope.
 (a) $25, \frac{1}{5}$ (b) $5, \frac{1}{25}$ (c) $25, \frac{1}{25}$ (d) $5, \frac{1}{5}$
3. The location of Sun in the sky, is the farthest from the celestial equator
 (a) in winter solstice and summer solstice (b) in vernal equinox and autumnal equinox
 (c) they coincide at all times (d) at noon each day of the year
4. The Sun's core contains about 10^{26} particles per cubic centimeter at a temperature of 15 million K. on the other hand, Earth's atmosphere at sea level has a density of about 2.4×10^{19} particles per cubic centimeter and the temperature is about 300 K. If we compare the pressure at Sun's core and Earth's atmospheric pressure at sea level the ratio becomes
 (a) 2×10^3 (b) 2×10^6 (c) 2×10^9 (d) 2×10^{11}
5. Sirius, the brightest star in our sky, has a measured parallax angle of $0.379''$. Its distance in parsecs is
 (a) 0.379 pc (b) 3.379 pc (c) 2.64 pc (d) 0.264 pc
6. The Sun's orbital velocity is 220 km/s and it is at distance of 27,000 light-years from the centre of the galaxy. Assuming simple Keplerian motion, the mass of the Milky Way Galaxy enclosed within the Sun's orbit is about
 (a) 10^{60} kg (b) 10^{41} kg (c) 10^{55} kg (d) 10^{82} kg
7. A visible-light emission line of hydrogen has rest wavelength of 656.3 nm, but it appears at 662.9 nm, in the spectrum of a distant galaxy. How fast is the galaxy moving away from us?
 (a) 300 km/s (b) $\frac{1}{3000}$ km/s (c) 3000 km/s (d) 30000 km/s
8. The orbital period of a geosynchronous satellite is equal to Earth's rotation period. The orbital radius of such a geosynchronous satellite in a circular orbit is about
 (a) 42,000 km (b) 35,800 km (c) 42,220 km (d) 42,180 km
9. Assume that you are looking at this question paper using the Hubble space telescope which has an angular resolution of 0.05 arc seconds, how far away could you place the question paper and still be able to read it? You would need to resolve at least 0.2 mm to read the letters clearly.
 (a) 8250 km (b) 825 m (c) 2850 km (d) 82 km

10. For very hot stars, the higher the temperature, the weaker are the hydrogen lines in their emission spectra. The reason is
- Strong helium lines cover the hydrogen lines
 - The hydrogen is used to form molecules
 - Very hot stars have converted hydrogen to helium
 - Too much of the hydrogen is ionized
11. The half circle in the sky, running precisely overhead between the celestial north pole and the celestial south pole is called the meridian. At any given location, the time between two successive meridian crossings (from east to west) of a star other than the Sun isthe time between two successive meridian crossings of the Sun.
- is always equal to
 - is always less than
 - is always greater than
 - is varying compared to
12. At the end stages of the very massive stars, the final element that is formed at their core, after which no further stable fusion of elements proceeds, is
- Uranium
 - Plutonium
 - Iron
 - Nickel
13. A polar satellite is revolving at a height of 600 km from the surface of the Earth (radius 6400 km). It passes over zenith of an observer. The angular displacement the observer will see in 10 seconds is
- 0.04°
 - 0.62°
 - 3.7°
 - 7.2°
14. A dwarf galaxy, of mass 10^8 solar masses, is in orbit around its parent galaxy. The orbital radius is 30 kpc. From an analysis of its spectrum the dwarf galaxy's tangential speed in its orbit is found to be 300 km/s. How many times smaller is the dwarf galaxy's mass compared to the amount of mass of the parent galaxy encircled by the orbit of the dwarf galaxy?
- the two masses are comparable
 - 10^4
 - 6250
 - 6000
15. Titius-Bode type relations give mutual relative distances of the planets around a parent star and also of the moons around a parent planet. One such function relation found for an exoplanetary system is
- $$a_n = 0.0142 e^{0.9975 n}$$
- Here n is the order of the planet when the distances, of the planets of a system from their parent star are put in ascending order. For a particular exoplanetary system the distance to the fifth planet works out as 2 AU. At what distance (in AU) would the tenth planet be from the star?
- 70.42
 - 14.19
 - 140.84
 - 281.69
16. The Sun rotates differentially, with the equatorial regions rotating faster compared to the regions closer to the poles. The equatorial regions complete one rotation in 24 days. Given the Sun's equatorial radius as 6.96×10^8 m, the observed difference in wavelength, between the two edges of the Sun along its equator, as observed in the oxygen 630 nm line is
- 0.0044 nm
 - 0.1 nm
 - 0.0088 nm
 - 0.2 nm
17. The relation R on Z , the set of integers, defined by $(x, y) \in R$ if and only if $x + y \neq 0$ is
- Reflexive but neither symmetric nor transitive
 - Symmetric but neither reflexive nor transitive
 - Transitive but neither reflexive nor symmetric
 - Neither reflexive nor symmetric nor transitive
18. A is a 23×23 skew symmetric matrix. What is the determinant of A ?
- 0
 - 2
 - 3
 - 23

19. For what values of k does the following system of linear equations have a non-zero solution?
- $$\begin{aligned} kx + y + z &= 0 \\ x + ky + z &= 0 \\ x + y + kz &= 0 \end{aligned}$$
- (a) $k = -1, k = 0$ (b) $k = 1, k = -2$ (c) $k = 3, k = 4$ (d) $k = 4, k = 5$
20. The number of real values of x that satisfy the equation $e^{2x} + e^x - 2 = 0$ is
- (a) 0 (b) 1 (c) 2 (d) 3
21. If z is a complex number such that $\left| \frac{z-2}{z+2} \right| = 1$, then which of these is not necessarily correct?
- (a) Real part of $z = 0$ (b) Imaginary part of $z = 0$
(c) $z + \bar{z} = 0$ (d) The only such real z is 0
22. The number of positive divisors of 194700 is
- (a) 6 (b) 18 (c) 36 (d) 72
23. p and q are positive real numbers. For what value of $\frac{p}{q}$ is the following function continuous at $x = 0$?
- $$f(x) = \frac{\sin px}{qx} \text{ if } x \geq 0 \text{ and } f(x) = 1 + \frac{\sin qx}{px} \text{ if } x < 0$$
- (a) $\frac{1 + \sqrt{5}}{2}$ (b) $\frac{1 - \sqrt{5}}{2}$ (c) $\frac{\sqrt{5} - 1}{2}$ (d) $\frac{\sqrt{5} - 2}{2}$
24. The value of the integral $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$ is
- (a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π
25. The area bounded by $y = 4 - x^2$ and $y = x^2 - 4$ is
- (a) $\frac{16}{3}$ (b) $\frac{32}{3}$ (c) $\frac{48}{3}$ (d) $\frac{64}{3}$
26. Five unbiased coins are tossed simultaneously. What is the probability that exactly two heads show up?
- (a) $\frac{5}{10}$ (b) $\frac{5}{16}$ (c) $\frac{5}{32}$ (d) $\frac{5}{64}$
27. The differential equation $\frac{dy}{dx} + \frac{x}{y} = 0$ represents
- (a) a family of congruent circles with centres on X-axis
(b) a family of congruent circles with centres on Y-axis
(c) a family of concentric circles centred at origin
(d) a family of concurrent straight lines
28. The point on the curve $y = x^2 + 3x + 4$ in the first quadrant at which the tangent passes through the origin is
- (a) (1, 8) (b) (2, 14) (c) (3, 22) (d) (4, 32)

29. Let $P(x)$ be a polynomial having non-negative integer coefficients such that the product of its coefficients is 73 and sum of the coefficients is 2023. What is its degree?
 (a) 1949 (b) 1950 (c) 1951 (d) 1952
30. How many positive integers less than 10000 have the digit 2, 4 or 6 in the hundred's place?
 (a) 1512 (b) 2700 (c) 3000 (d) 3600
31. The geographical coordinates of the peak of Mount Everest are $27^\circ 59' 18''$ N and $86^\circ 55' 31''$ E. Which of the following relations may be used to determine approximately the latitude ℓ of an observer, who, standing on the same longitude as that of the peak of Everest and at mean sea level will see the pole star in line with the peak? [The curvature of Earth has been ignored; h stands for the height of Mount Everest from sea level, g stands for the longitude of the peak of Mount Everest and R_{Earth} stands for the radius of Earth.]
- (a) $\ell_{Everest} - \frac{h \tan \ell}{R_{Earth}} = \ell$ (b) $\ell_{Everest} - \frac{h \cot \ell}{R_{Earth}} = \ell$
 (c) $\ell_{Everest} - \frac{h \sin g}{R_{Earth}} = \ell$ (d) $\ell_{Everest} - \frac{h \cos \ell}{R_{Earth}} = \ell$
32. A businessperson buys 11 items for 10 rupees and sells them all at the rate of rupees 11 per 10 items. The whole lot is bought back for rupees 12 per item and re-sold at the rate of rupees 13 per item. The correct statement is
 (a) Profit from the second sale is equal to the profit from the first sale
 (b) Percentage profit from the second sale is larger than the percentage profit from the first sale
 (c) Profit from the second sale is larger but the profit percentage is the same as in the first case.
 (d) The percentage profit from the first sale is 21%
33. Two moles of ideal gas at 1 bar pressure are compressed isothermally to half its volume at 300 K. It is then brought to original pressure in an isochoric process. Further it is expanded to original volume in an isobaric process. The total amount of heat given out by the system in the cycle is
 (a) 963 J (b) 2494 J (c) 3458 J (d) 5952 J
34. Effective solar power incident on the Earth is about 1350 W/m^2 . A converging lens of 100 mm aperture diameter with focal length of 50 mm is used to focus sunlight on a paper, on a bright sunny day. As seen from Earth the angular diameter of the Sun is observed as $\frac{1}{2}$ a degree. The estimated amount of power per unit area (in W/m^2) in the image is (assuming lens allows 85 % energy to pass through it):
 (a) 1.1×10^7 (b) 4.5×10^7 (c) 6.0×10^7 (d) 2.8×10^8
35. A symmetric biconvex lens has focal length of 40 mm in air. It is kept on the surface of water in a tank. The focal length inside the water is now 70 mm. If refractive index of water is $\mu_w = \frac{4}{3}$, The refractive index of the material of the lens is
 (a) 1.7 (b) 1.8 (c) 1.9 (d) 2.0

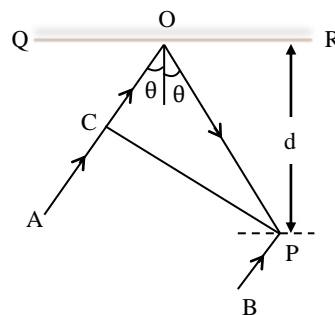
36. A parallel plate capacitor, with air in gap, is charged to 10 V. If now it is filled with material having dielectric constant 2.50, which of the following is possibly correct?
- (a) Potential difference remains 10 V but charge reduces by factor of 2.50
 (b) Potential difference becomes 25 V and charge increases by factor of 2.50
 (c) Potential difference becomes 4.0 V and charge increases by factor of 2.50
 (d) Potential difference becomes 4.0 V and charge remains the same.
37. The heat given to the ideal gas system in the following cycle, if at lower left end of cycle the pressure is 1.00 bar is approximately



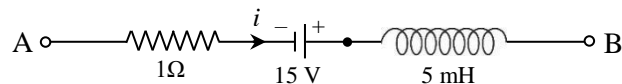
- (a) 19 J (b) 24 J (c) 274 J (d) 337 J
38. When a current of 14.0 A flows through a circular coil of radius 10.0 cm, the magnetic field at its centre is twice the horizontal component of Earth's magnetic field at a particular location. When the coil is placed in a vertical plane perpendicular to magnetic meridian at the same location and the current is passed, the angle of dip at that point becomes half compare to that before the current starts. The magnitude of earth's magnetic field at that location is
- (a) 0.88 gauss (b) 0.51 gauss (c) 0.44 gauss (d) 0.25 gauss

39. In the adjacent diagram, CP represents a wave front and AO and BP, the corresponding two rays. Find the condition of θ for constructive interference at P between the rays BP and reflected ray AOP:

(a) $\cos \theta = \frac{3\lambda}{2d}$ (b) $\cos \theta = \frac{\lambda}{4d}$
 (c) $\sec \theta - \cos \theta = \frac{\lambda}{d}$ (d) $\sec \theta - \cos \theta = \frac{4\lambda}{d}$



40. The network shown in the figure is a part of a complete circuit. If at a certain instant the current i is 5 A and it is decreasing at the rate of 10^3 A/s then $V_B - V_A$ is



- (a) 5 V (b) 10 V (c) 15 V (d) 20 V
41. A particle of mass m is moving in a circular path of radius 'r' such that its centripetal acceleration is $a_c = Krt^2$ where K is a constant.
- (a) The speed of the particle at time t is $v = Krt$
 (b) The speed of the particle on the circular track remains same throughout
 (c) The power delivered to the particle during the course of its motion is $P = mKrt^2$
 (d) The force responsible to this kind of motion is $F = mKr$

42. An incompressible and non-viscous liquid of density 1.25 g/cm^3 flows with a speed of 25 cm/s through a horizontal pipe of diameter 7.0 cm . It has a constriction where diameter reduces to 5.0 cm . The pressure change at the constriction is
 (a) -150 Pa (b) -111 Pa (c) -625 Pa (d) -375 Pa
43. Two protons collide head-on with energy of 100 keV each. The minimum distance they can approach is
 (a) 7.2 fm (b) 14.4 fm (c) 14.4 pm (d) 28.8 pm
44. The force of interaction between two atoms is given by $F = PQ \exp\left(\frac{-x^2}{Pk_B T}\right)$; where x is the distance, k_B is the Boltzmann constant, T is temperature, and P, Q are two constants. The dimensions of P and Q are
 (a) MT^{-2} and M^0LT^{-3} (b) M^{-1}L^2 and $\text{M}^2\text{L}^{-1}\text{T}^{-4}$
 (c) M^{-1}T^2 and M^2LT^{-4} (d) M^0LT^{-2} and $\text{M}^{-1}\text{L}^{-3}\text{T}^{-1}\text{K}^2$
45. A small plane mirror is placed at the center of a spherical screen of radius 4 cm . A beam of light is falling on the mirror. If the mirror is rotated about an axis lying in the plane of mirror and perpendicular to the incident ray so as to make N revolutions per second, the speed of the spot of light on the screen after reflection from the mirror will be
 (a) $\frac{N}{\pi}$ units (b) $\frac{2N}{\pi}$ units (c) $16N\pi$ units (d) $8N\pi$ units
46. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 1136 nm is incident on it. The band gap (in eV) of the semiconductor is
 (a) 0.71 (b) 1.09 (c) 0.80 (d) 1.31
47. A non-conducting cube of side 25.0 cm has identical small circular conducting loops of radii 10.0 cm and 100 turns, one on each surface, centred at the centre of the surface. The surfaces of the cube are parallel to XY, YZ and ZX planes. Each loop carries 100 mA current such that net magnetic field at the centre of the cube is zero. In one loop on a surface parallel to XY plane, the direction of current is reversed, the magnetic field (in milli gauss) at the centre of the cube will then be
 (a) Zero (b) 306 (c) 480 (d) 610
48. Radio signals are added to the signal generated by a local oscillator (LO) to get an Intermediate Frequency (IF) signal. The LO has a predefined frequency range from ν_L MHz to ν_U MHz. In the Giant Meter Wave Radio Telescope (GMRT) at Pune, radio waves ranging in frequency from $f_L = 30 \text{ MHz}$ to $f_U = 1660 \text{ MHz}$ are added to the output from a LO to produce an IF signal centred at 70 MHz . For the LO, it is good to keep the ratio $\frac{\nu_U}{\nu_L}$ as small as possible. For this, ν_L and ν_U should be such that
 (a) $\nu_L > f_L ; \nu_U < f_U$ (b) $\nu_L > f_L ; \nu_U > f_U$
 (c) $\nu_L < f_L ; \nu_U < f_U$ (d) $\nu_L < f_L ; \nu_U > f_U$

ANY NUMBER OF OPTIONS (4, 3, 2 or 1) MAY BE CORRECT
MARKS WILL BE AWARDED ONLY IF ALL THE CORRECT OPTIONS ARE BUBBLED AND NO INCORRECT

49. Consider a binary star system of star A (a hot star) and star B (a colder star) at edge-on configuration. In such a system eclipses occur when one star comes in front of the other in our line of sight. The correct statement(s) is/are
- A shallower dip in the emission occurs when B is eclipsing (passing in front of) star A
 - A shallower dip in the emission occurs when the star A is eclipsing star B
 - The duration from start of eclipse to the minimum of the deeper dip in the binary emission is longer if star B has the smaller radius
 - The duration of the stronger/deeper dip is longer if star A has the larger radius
50. The position of a particular star in the sky with respect to an observer depends on
- the right ascension of the star
 - the declination of the star
 - latitude of the observer
 - longitude of the observer
51. The typical spectrum of a star showing blackbody emission along with absorption lines gives information about
- temperature
 - composition
 - luminosity
 - mass
52. A rocky exoplanet having radius r_p is on a circular orbit of radius r_o around its parent star. The parent star has luminosity L and its radius is r_s . The system is at a distance D from the observer. The planet has no atmosphere. Approximately how much area on the star's surface will the planet block from view when it comes in the line of sight to the star?
- $\pi r_p^2 \left(1 + \frac{2}{D}(r_o - r_s)\right)$
 - $\pi r_s^2 \left(1 + \frac{r_o}{D}\right)$
 - $\pi r_o r_p \left(1 + \frac{r_o - r_s}{D}\right)$
 - $\pi r_p^2 \left(1 + \frac{r_o - r_s}{D}\right)$
53. Which of the following is/are true for the function $f(x) = x^3|x|$, $x \in R$? Here R is the set of real numbers.
- f is differentiable at 0
 - f is not differentiable at 0
 - f is three times differentiable at $x = 0$ but not four times
 - f is discontinuous at $x = 0$
54. If a, b, c are the roots of the equation $x^3 - 7x^2 - 2023x - 1 = 0$, then which of the following is/are true? It is given that the discriminant is greater than zero.
- $ab + bc + ca = -2023$
 - $(a^2 + b^2 + c^2) \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \right) \geq 7$
 - $(a^4 + b^4 + c^4) \left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{1}{c^4} \right) \geq 4$
 - $a + b + c > 7$

55. Consider the LPP

Max $Z = 2x + 3y$ subject to the constraints

$5x + 4y \geq 20$, $y - x \leq 1$, $x - 2y = 4$, $x, y \geq 0$

Which of the following is/are true?

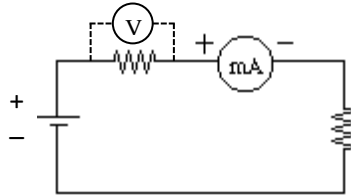
- (a) $x = 4, y = 1$ is a feasible solution of the LPP.
- (b) $x = 6, y = 1$ and $x = 24, y = 10$ are feasible solutions of the LPP.
- (c) $x = 4 + 2k, y = k$ is a feasible solution of the LPP for any real number k .
- (d) The LPP has an unbounded optimal solution.

56. Consider the relation R on a subset A of real numbers defined as $(x, y \in R)$ if and only if $xy \neq 0$.

Which of the following is/are false?

- (a) R is an equivalence relation on any such set A
- (b) R is transitive on any such set A
- (c) R is an equivalence relation on $A = (0,1)$
- (d) R is an equivalence relation on $A = (-2023, 2023)$

57. Consider the following circuit. The battery terminal voltage is 3.60 V. The two resistors are 3300Ω and 300Ω . When the voltage across each resistor is measured with a voltmeter, it showed 3.20 V and 0.291 V respectively. Consider Ammeter resistance to be negligible. The correct statement(s) is/are



- (a) the resistance of the voltmeter is 8800Ω
- (b) the current in the ammeter when volt meter is connected to 3300Ω is 1.33 mA
- (c) the current in the ammeter when the voltmeter is connected across 300Ω is more than 1 Amp
- (d) the current in the ammeter when voltmeter is not connected is 1 mA

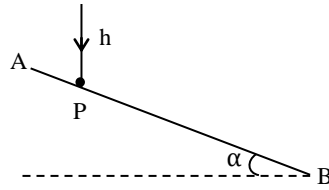
58. A ball P moving with speed 20 ms^{-1} collides elastically with another identical ball Q. At the instant of the collision, P was moving at an angle of 60° from the line joining the centers of the two balls. Select the correct option(s)

- (a) After the collision the two balls move perpendicular to each other
- (b) After the collision the speed of ball P is $10\sqrt{3} \text{ ms}^{-1}$
- (c) After the collision the speed of ball Q is 10 ms^{-1}
- (d) Kinetic energy will not be conserved here

59. The optical powers of the objective and the ocular (eye piece) of a microscope are equal to 100 and 20 D respectively. The microscope magnification is equal to 50 for relaxed eye.

- (a) The object is located at 1.1 cm in front of the objective
- (b) The length of the microscope tube (separation between objective and eyepiece) is 16 cm
- (c) The magnification of the microscope becomes 60 when the distance between the objective and the ocular i.e the tube length is increased by 2.0 cm.
- (d) The magnifying power is 62 if the final image is formed at infinity for the original arrangement prior to increasing the tube length

60. Having fallen vertically under gravity through a height 'h' a small solid ball rebounds elastically off a rigid and smooth inclined plane AB forming an angle α with horizontal.



- (a) The ball will rebound second time, off the inclined plane, at a distance of $S = 8h \sin \alpha$ along the incline from the point P of first impact.
- (b) The time taken between the first impact to the second impact on the incline is $t_1 = \sqrt{\frac{8h}{g}}$
- (c) The speed of the ball just after the first impact is $\sqrt{2gh}$
- (d) After the first impact with the inclined plane the ball will move at an angle of $90 - 2\alpha$ with the horizontal

Rough Work