IOQP 2021-22 PART I (NSEP – 2021-22) Solution– 61

1. The instantaneous rate of absorption of heat is $\frac{dQ}{dt} \propto 4\pi r^2 = k4\pi r^2$ Also $\frac{dQ}{dt} = -L\frac{dm}{dt}$ So $-L\frac{dm}{dt} = k4\pi r^2$ *or* $-L\frac{d}{dt}\left(\frac{4\pi}{3}r^3\rho\right) = k4\pi r^2$ or $\frac{dr}{dt} = -\frac{k}{\rho L} \Rightarrow \int_{r_0}^{r} dr = -\frac{k}{\rho L}\int_0^t dt \Rightarrow r = r_0 - \frac{k}{\rho L}t$ or $r = -\frac{k}{\rho L}t + r_0$ which is a straight line with negative slope $= -\frac{k}{\rho L}$ where k is constant. **Ans: c**

2. The process from A to B is isochoric $\because P \propto T$ means the volume is constant. Therefore the work done $dW_{AB} = PdV = 0$ From B to C the process is isobaric so work done is $dW_{BC} = PdV = nRdT = 3\times R\times(600-200) = 1200R$ .CD is again isochoric process so work done $W_{CD} = 0.$ Further the process from D to A is isobaric means P constant and work done is $dW_{DA} = PdV = nRdT$ or $dW_{DA} = 3\times R(100-300) = -600R.$ Thus the total work done is $W = 1200R - 600R = 600R = 4986J = 4.986kJ = 5.0kJ$ **Ans: b**

3. From first law of thermodynamics $dQ = dU + dW \Rightarrow C\,dT = C_V\,dT + P\,dV.$ Given that $C = C_V + \alpha\frac{P}{T}$ Substituting the value we get $\left(C_V + \alpha\frac{P}{T}\right)dT = C_V dT + PdV \Rightarrow \alpha\frac{dT}{T} = dV$ on integration we get $\alpha\times\ell nT = V + \text{constant}$ or $T = Ae^{V/\alpha}$ **Ans: d**

4. The magnetic field produced by a current carrying conductor at a distance x is $B = \frac{\mu_0 I}{2\pi x}$
Therefore the induced emf in a conductor of length dx moving with velocity v is
$d\varepsilon = -\ell Bv = -\frac{\mu_0 I}{2\pi x}v\,dx.$ Total emf produced in the present problem is
$$\varepsilon = -\int_{2\ell}^{3\ell}\frac{\mu_0 I}{2\pi x}v\,dx = -\frac{\mu_0 I}{2\pi}v\int_{2\ell}^{3\ell}\frac{dx}{x} = \frac{\mu_0 I}{2\pi}v\times\ln 1.5$$
Ans: d

5. Since both the charges are positive, the electric field at any point between them is $E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{x^2} - \frac{Q}{(L-x)^2}\right]$ This will be positive for $0 < x < \frac{L}{2}$ and negative for $\frac{L}{2} < x < L$ as shown in figure d. The curve is a $\frac{1}{x^2}$ type. **Ans d**

6. When the current flows in the wire along AB, the magnetic field in the circular loop is directed outward perpendicular to the plane of the paper. During the increase of current i.e., from 0 to T/2, the induced current in the loop is clockwise while during the decrease of current i.e., T/2 to T the induced current shall be anticlockwise. Hence the answer is a.
Ans a

7. The $2.0 M\Omega$ resistance is connected in series with R_B and the cell. When we connect the voltmeter of resistance $r\, M\Omega$ in parallel to $2.0 M\Omega$ we get $3 = \frac{9}{\frac{2r}{r+2} + R_B} \times \frac{2r}{r+2}$

$\Rightarrow 2r + R_B(r+2) = 6r \Rightarrow R_B = \frac{4r}{r+2}$ When we connect the same voltmeter in parallel with R_B we get $4.5 = \frac{9}{2 + \frac{rR_B}{r+R_B}} \times \frac{rR_B}{r+R_B} \Rightarrow 2(r + R_B) + rR_B = 2rR_B \Rightarrow R_B = \frac{2r}{r-2}$ Comparing the result

$\frac{4r}{r+2} = \frac{2r}{r-2} \Rightarrow r = 6$ Thus $r = 6 M\Omega$ and $R_B = 3\, M\Omega$ **Ans: a**

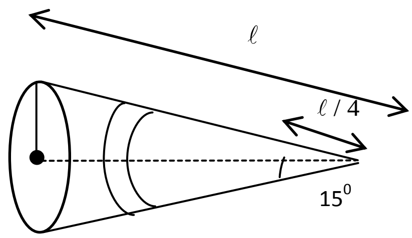
8. The magnifying power of a compound microscope when the final image is formed at D, the least distance of distinct vision is $MP = \frac{V_0}{U_0}\left(1 + \frac{D}{f_e}\right)$ Now as per the given conditions

$50 = \frac{V_0}{U_0}\left(1 + \frac{25}{5}\right) \Rightarrow \frac{V_0}{U_0} = \frac{25}{3}$ Now for the objective lens $\frac{1}{V_0} - \frac{1}{-U_0} = \frac{1}{f_0} \Rightarrow 1 + \frac{V_0}{U_0} = \frac{V_0}{f_0}$

comparing the two, we get $V_0 = \frac{28}{3} cm.$ Increasing the length of microscope by 2 cm, Then

$V_0 = \frac{28}{3} + 2 = \frac{34}{3} cm$ In the new situation $1 + \frac{V_0'}{U_0'} = \frac{V_0'}{f_0} = \frac{34}{3} \Rightarrow \frac{V_0'}{U_0'} = \frac{31}{3}$ The magnifying power therefore now becomes $MP = \frac{31}{3}\left(1 + \frac{25}{5}\right) = 62$ **Ans: a**

9. Let us consider an elementary ring of width $d\xi$ at a slant distance ξ from the vertex of the cone. The charge on the circular ring shall be $dq = 2\pi\xi \sin\theta\, d\xi \times \sigma$. The electric field produced by this elementary ring at the vertex of the cone is $dE = \frac{1}{4\pi\varepsilon_0} \times \frac{2\pi\xi \sin\theta\, d\xi \times \sigma \times \xi\cos\theta}{\xi^3}$ Thereby the electric field



E at the vertex shall be $E = \frac{\sigma}{4\varepsilon_0} 2\sin\theta\cos\theta \int_{l/4}^{l} \frac{d\xi}{\xi} \Rightarrow E = \frac{\sigma}{4\varepsilon_0}\sin 2\theta \{\ln\xi\}_{l/4}^{l}$

$\Rightarrow E = \frac{\sigma}{8\varepsilon_0} \times 2\ln 2 = \frac{\sigma}{4\varepsilon_0}\ln 2$ **Ans: b**

10. According to the Newton's Second Law $F = \frac{dp}{dt}$. In the present case the rain drop is attracted by the earth so at any instant, $mg = \frac{d}{dt}(mv) \Rightarrow mg = m\frac{dv}{dt} + v\frac{dm}{dt}$ *or* $g = \frac{dv}{dt} + \frac{v}{m}\frac{dm}{dt}$

Given that $\frac{dm}{dt} = kt^2 \Rightarrow m = \frac{kt^3}{3} + m_0$ where m_0 is initial mass. Further

$$g = \frac{dv}{dt} + \frac{v}{\frac{kt^3}{3} + m_0} \times kt^2 \Rightarrow \frac{dv}{dt} = g - \frac{3kt^2}{3m_0 + kt^3} \times v \Rightarrow \frac{dv}{dt} + \left(\frac{3kt^2}{3m_0 + kt^3}\right)v = g \text{ or}$$

$$\Rightarrow \left(3m_0 + kt^3\right)dv + v3kt^2 dt = g\left(3m_0 + kt^3\right)dt \text{ or} \Rightarrow d\left\{\left(3m_0 + kt^3\right)v\right\} = g\left(3m_0 + kt^3\right)dt$$

Integrating we get $\left(3m_0 + kt^3\right)v = \left(3m_0 gt + \frac{gkt^4}{4}\right)_0^t$ or

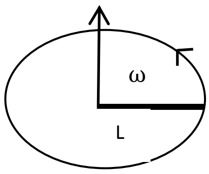
$$v = g\frac{12m_0 t + kt^4}{4\left(3m_0 + kt^3\right)} = \frac{1905}{1506}g = 12.4\, ms^{-1}$$ **Ans: a**

11. Two planets of mass M and radius R each are separated by distance 4R. A mass m has to be thrown from Planet A so as just to reach Planet B. For this we need to throw the mass so that it just reaches the midpoint then after it will be attracted by B. The potential energy of the mass m on the surface of the planet A is $U_A = -\frac{GMm}{R} - \frac{GMm}{3R} = -\frac{4GMm}{3R}$ and the potential energy at the midpoint between the two planets is $U_{Mid} = -\frac{GMm}{2R} - \frac{GMm}{2R} = -\frac{GMm}{R}$

Hence the energy needed to project the body is

$$\frac{1}{2}mv^2 = \left(-\frac{GMm}{R}\right) - \left(-\frac{4GMm}{3R}\right) \Rightarrow v = \sqrt{\frac{2GM}{3R}}$$ **Ans: b**

12. When the rod rotates about a vertical axis through one of its ends, every pointon the rod experiences a centrifugal force. If we consider a small length dx of mass $\lambda\, dx$ at a distance x from the axis where $\lambda = \frac{M}{L} = \frac{\pi R^2 L\rho}{L} = \pi R^2 \rho$,



The outward pull on this length x is $= \frac{\lambda\, dx\, \omega^2\, x^2}{x} = T\left(say\right)$

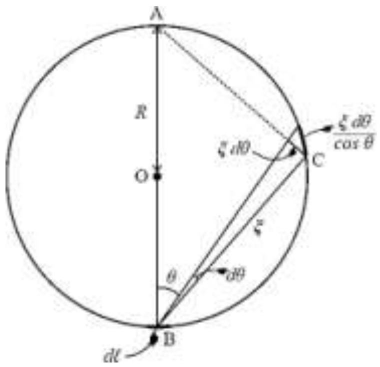
This force will cause an elongation in the rod, because of its elasticity.

The elongation may be given by $d\xi = \frac{T\,x}{AY} = \frac{\lambda\, dx\, \omega^2\, x}{\pi R^2 Y} . x = \frac{\rho\omega^2}{Y} . x^2\, dx$.

The total elongation in the rod is therefore $\int d\xi = \frac{\rho\omega^2}{Y}\int_0^L x^2\, dx = \frac{\rho\omega^2 L^3}{3Y}$.

Ans: b

13. Since the total energy is fixed, the kinetic energy so to say the magnitude of the momentum will be large where ever the potential energy is less and vice versa. Further the momentum $p = \pm\sqrt{2m\times(KE)} = \pm\sqrt{2m\times(E-V)}$. Here (E-V) is the kinetic energy of the particle. The curve for momentum will be symmetric about x axis so curve a.
Ans: a



14. In a hollow metallic cylinder current is sent parallel to its axis along the entire curved surface. Let us consider a thin strip of width dl on the surface and along the length of the cylinder at a point B and another parallel strip of width $\frac{\xi d\theta}{\cos\theta}$ at the end of the chord of length ξ at angle θ . If I be the current through the metallic cylinder then the current per unit width shall be $j = \frac{I}{2\pi R}$. Thereby the current through the two parallel strips separated by ξ shall be $jdl\ and\ j\frac{\xi d\theta}{\cos\theta}$ respectively.

The force of attraction between these two parallel wires shall thus be $F = \frac{\mu_0}{2\pi}\frac{jdl\times j\xi d\theta}{\xi\cos\theta}\ Nm^{-1}$. Component of this force towards the centre will add togive resultant inward force then dividing by dl and integrating over θ within the limits $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$ we obtain force per unit surface of the hollow cylinder. Thusthe inward force per unit surface area is

$$=\sum F\cos\theta = \frac{\mu_0 j^2}{2\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\theta = \frac{\mu_0 j^2}{2} = \frac{\mu_0}{2}\left(\frac{I}{2\pi R}\right)^2 = 2.55\times10^{-7}\,Nm^{-2}$$

Ans: d

15. By the principle of conservation of momentum along x-direction $2Mv\cos\theta + M\frac{3}{2}v\cos\phi = 2M\times\frac{4}{5}v + MV_X$ where V_x is the velocity of mass M in x direction after the collision. Substituting the values $2Mv\times\frac{4}{5} + M\frac{3}{2}v\times\frac{3}{5} = 2M\times\frac{4}{5}v + MV_X \Rightarrow V_X = \frac{9}{10}v$

In y-direction the momentum conservation yields $-2Mv\sin\theta + M\frac{3}{2}v\sin\phi = 2M\times 0 + MV_y$

substituting the values $-2Mv\frac{3}{5} + M\times\frac{3}{2}v\times\frac{4}{5} = 0 + MV_y \Rightarrow V_y = 0$ means no velocity in y-direction. Hence the centre of mass moves along x-direction. The velocity of centre of mass after the collision is $V_{xCM} = \frac{2M\times\frac{4}{5}v + M\times\frac{9}{10}v}{3M} = \frac{25}{30}v = \frac{5}{6}v$ and not $\frac{5}{2}v$ Also the linear momentum before collision is $\frac{25}{10}Mv = \frac{5}{2}Mv$ and not $\frac{5}{6}Mv$ **Ans: b**

16. Let us consider that the height of the liquid surface in the hemispherical bowl is h at a certain time t. The radius of water surface at this time shall be $=\sqrt{R^2-(R-h)^2}$ So the surface area of the liquid at this time will be $=\pi\left\{R^2-(R-h)^2\right\}=\pi\left(2Rh-h^2\right)$ Further considering that the liquid height falls through dh in time dt, the volume of liquid flowing out per second can be written as $-\pi\left(2Rh-h^2\right)\frac{dh}{dt}=va=a\sqrt{2gh}$ Thereby $dt=-\frac{\pi}{a\sqrt{2g}}\left(2Rh^{1/2}-h^{3/2}\right)dh$ integrating we get $\int_0^t dt=-\frac{2\pi R}{a\sqrt{2g}}\int_{R/2}^{0} h^{1/2}dh+\frac{\pi}{a\sqrt{2g}}\int_{R/2}^{0} h^{3/2}dh \Rightarrow t=\frac{17\pi R^2}{60a}\sqrt{\frac{R}{g}}$

Ans: c

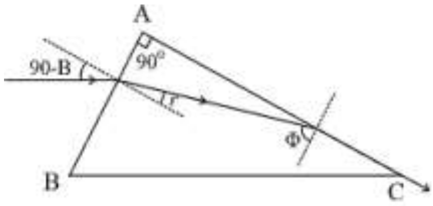
17. To obtain dimensional formula for ε_0 let us express Coulomb's law as

$$F=\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{r^2}=4\pi\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\times\frac{q_1}{r}\right)\left(\frac{1}{4\pi\varepsilon_0}\times\frac{q_2}{r}\right)\Rightarrow\left(\frac{F}{4\pi r^2}\right)r^2\cong\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\times\frac{q_1}{r}\right)\left(\frac{1}{4\pi\varepsilon_0}\times\frac{q_2}{r}\right)$$

$\Rightarrow Pa\times L^2=\varepsilon_0V^2\Rightarrow\varepsilon_0=(Pa)^1L^2V^{-2}$ **Ans: b**

18. The observer will record the maximum frequency $\upsilon'=\frac{v}{v-v_s}\times\upsilon$ when the sound produced by the siren, in the top most point of the circumference of the wheel, reaches the listener. This sound will reach the listener in time $t=\frac{100}{330}=0.303\text{s}$ after being produced. Also the wheel itself will take time $t_0=\frac{3}{4}\times\frac{2\pi}{\omega}$ substituting $\omega=\pi$ we get $t_0=\frac{3\pi}{2\pi}=1.5\ s$ Hence the total time is $t+t_0=0.303+1.5=1.803s$ **Ans: b**

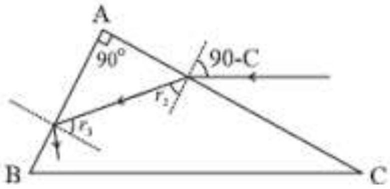
19. For a ray of light incident on side AB parallel to the base, we can write that the refractive index



$$\mu = \frac{\sin(90-B)}{\sin r} = \frac{\cos B}{\sin r} \text{ or } \mu \sin r = \cos B \text{....(1)}$$

Also $r+\phi = 90$ or $\sin r = \sin(90-\phi) = \cos\phi = \sqrt{1-\sin^2\phi}$

or $\sin r = \sqrt{1-\frac{1}{\mu^2}} = \sqrt{\frac{\mu^2-1}{\mu^2}}$ or $\mu \sin r = \sqrt{\mu^2-1}$ ...(2)



From (1) and (2) $\cos B = \sqrt{\mu^2-1}$

or $\cos^2 B = (\mu^2-1)$....(3)

Next the ray is incident parallel to base on the side AC.

Here also $r_2 + r_3 = 90$ and since $r_3 > \phi$ i.e.the critical angle Now using $r_2 = 90 - r_3$ and

$\mu = \frac{\sin(90-C)}{\sin r_2} = \frac{\sin(90-C)}{\sin(90-r_3)} = \frac{\cos C}{\cos r_3}$ or $\mu \cos r_3 = \cos C = \cos(90-B) = \sin B$ This in turn gives $\mu^2(1-\sin^2 r_3) = \sin^2 B$ .... (4) adding equation (3) and (4)

$$\left(\mu^2 - \mu^2\sin^2 r_3\right) + \mu^2 - 1 = 1 \Rightarrow 2\left(\frac{\mu^2-1}{\mu^2}\right) = \sin^2 r_3 \text{....(5)}$$

Further angle $r_3 > \phi$ or $\sin r_3 > \sin\phi$

$\Rightarrow \sqrt{2\left(\frac{\mu^2-1}{\mu^2}\right)} > \sin\phi \Rightarrow 2\left(\frac{\mu^2-1}{\mu^2}\right) > \frac{1}{\mu^2} \Rightarrow \mu^2 - 1 > \frac{1}{2} \Rightarrow \mu^2 > \frac{3}{2} \Rightarrow \mu > \sqrt{\frac{3}{2}}$ Also

$r + \phi = 90$ *but* $r < \phi$ So essentially $\phi > 45$ or $\sin\phi > \sin 45$ *or* $\frac{1}{\mu} > \frac{1}{\sqrt{2}} \Rightarrow \mu < \sqrt{2}$ Thus

we can conclude that $\sqrt{\frac{3}{2}} < \mu < \sqrt{2}$ **Ans: b**

20. Under the conditions of pure rolling of the disc, the velocity of the point A (at the top) on the circumference is $v + \omega R = 2v$ where as the velocity of point B at half the radius is

$v + \omega\frac{R}{2} = \frac{3}{2}v$ Let the final speed of point A becomes $\frac{3}{2}v$ after turning through an angle ϕ

then $\frac{3}{2}v = \sqrt{v^2 + \omega^2R^2 + 2v\omega R\cos\phi} = \sqrt{v^2 + v^2 + 2v^2\cos\phi} \Rightarrow \frac{3}{2} = \sqrt{2+2\cos\phi}$ or

$\Rightarrow \frac{3}{2} = \sqrt{2+2\left(2\cos^2\frac{\phi}{2} - 1\right)} = 2\cos\frac{\phi}{2}$ *or* $\cos\frac{\phi}{2} = \frac{3}{4} \Rightarrow \phi = 82.82^\circ$ Further if $T = \frac{2\pi R}{v}$ be the time period then by simple unitary method time taken to turn through $\phi = 82.82^\circ$ is

$t = \frac{\phi}{360} \times \frac{2\pi R}{v} = 0.036\ s$ **Ans: b**

21. At the point of closest approach (distance) the particle will have tangential velocity expressed as $v_t = \omega d = \dot{\theta} d$ By conservation of angular momentum $mvb = I\omega = md^2\dot{\theta} \Rightarrow \dot{\theta} = \omega = \frac{vb}{d^2}$
This being a case of elastic scattering, the conservation of energy provides
$\frac{1}{2}mv^2 + 0 = \frac{1}{2}md^2\omega^2 - \int \frac{K}{r^3}.\,dr \Rightarrow mv^2 = md^2\omega^2 + \frac{K}{d^2} \because PE = -\int \frac{K}{r^3}dr = \frac{K}{2r^2} = \frac{K}{2d^2}$
Thereby $mv^2 = md^2\left(\frac{vb}{d^2}\right)^2 + \frac{K}{d^2} = \left(mv^2b^2 + K\right)\frac{1}{d^2}$ Substituting $m = 10^{-10}\,Kg$,
$v = 10^5\,ms^{-1}$ and numerically $K = b^2$ we obtain $d^2 = \left(mv^2b^2 + k\right)\frac{1}{mv^2} = 2b^2 \Rightarrow d = b\sqrt{2}$
Ans: b

22. According to the law of radioactive disintegration $N = N_0 e^{-\lambda t}$ The activity therefore is $-\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$ Given that at certain time t the activity of the sample is $\left(-\frac{dN}{dt}\right)_t = \frac{3}{5}\left(-\frac{dN}{dt}\right)_{t=0} = \frac{3}{5}\lambda N_0$ So $e^{-\lambda t} = \frac{3}{5} \Rightarrow \lambda t = \ln\left(\frac{5}{3}\right)$ or $\frac{\ln 2}{T} \times t = \ln\left(\frac{5}{3}\right)$
$\Rightarrow t = \frac{T \times \ln(5/3)}{\ln 2} = \frac{5570 \times 0.5108}{0.693} = 4105\,years$ **Ans: b**

23. The statement I is false but the statement II is true hence **Ans: d**

24. The statement I is true and the statement II is also true. Also the statement II is the cause of I hence **Ans: a**

25. In a swinging simple pendulum, the tension in the string at any arbitrary position may be expressed as $T = mg\cos\theta + \frac{mv^2}{l}$ The conservation of energy provides
$\frac{1}{2}mv^2 = mg\left(l\cos\theta - l\cos\theta_m\right)$ thereby $\frac{mv^2}{l} = 2mg\left(\cos\theta - \cos\theta_m\right)$ therefore the tension becomes $T = mg\left(3\cos\theta - 2\cos\theta_m\right)$ Obviously the tension depends on the angle θ and will be maximum when $\theta = 0$ So the maximum tension is $T_{\max} = mg\left(3 - 2\cos\theta_m\right)$ and the minimum tension (when $\theta = \theta_m$) is $T_{\min} = mg\cos\theta_m$ According to the given condition
$T_{\max} = 3T_{\min}$ Hence $mg\left(3 - 2\cos\theta_m\right) = 3mg\cos\theta_m \Rightarrow \cos\theta_m = \frac{3}{5} \Rightarrow \theta_m = 53.13^0$
$\Rightarrow \frac{\pi}{4} \le \theta_m \le \frac{\pi}{3}$ and the maximum tension is $T_{\max} = \frac{9}{5}mg.$ The maximum velocity
$v_{\max}^2 = 2gl\left(1 - \cos\theta_m\right) = 2gl\left(1 - \frac{3}{5}\right) = \frac{4gl}{5} \Rightarrow v_{\max} = \sqrt{\frac{4gl}{5}} = 3.96\,ms^{-1}$ **Ans: a,b,c,d**

26. When the masses are released, they move in opposite direction with equal momentum i.e., $mv+MV=0 \Rightarrow V=-\frac{mv}{M}$ ...(1) Numerically $V=\frac{mv}{M}$ Let the two masses collide for the first time after time t and the mass m turns through angle θ during this period then $t=\frac{\theta}{\omega_1}=\frac{2\pi-\theta}{\omega_2}$ or $t=\frac{\theta R}{v}=\frac{(2\pi-\theta)R}{V} \Rightarrow \left(\frac{1}{v}+\frac{1}{V}\right)\theta=\frac{2\pi}{V}$ Substituting the value of V, we obtain $\theta=\frac{2\pi M}{m+M}=\frac{4\pi}{3}$ if M = 2m. Also Energy conservation provide that

$$\frac{1}{2}mv^2+\frac{1}{2}MV^2=U_0 \quad or \quad \frac{1}{2}mv^2+\frac{1}{2}M\left(\frac{mv}{M}\right)^2=U_0 \text{ or } \frac{1}{2}mv^2\left[1+\frac{m}{M}\right]=U_0$$

$\Rightarrow v=\sqrt{\frac{2MU_0}{m(\mathrm{m+M})}}=\sqrt{\frac{4U_0}{3m}}$ if M = 2m Thus the time taken for firstcollision is

$t=\frac{\theta R}{v}=\frac{4\pi R}{3\sqrt{\frac{4U_0}{3m}}}=2\pi R\sqrt{\frac{m}{3U_0}}$ Lastly the time taken for the second collision must be just double of it and not $2\pi R\sqrt{\frac{2m}{3U_0}}$ **Ans: a, b, c**

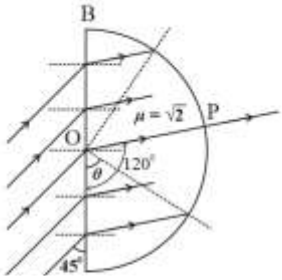
27. Given that the Electric field $\vec{E}=(3\hat{j}+b\hat{k})\times10^{-3}\sin[10^7(\mathrm{x}+2\mathrm{y}+3\mathrm{z}-\beta\mathrm{t})]$ Knowing$\vec{K}.\vec{r}=(\hat{\imath}k_x+\hat{\jmath}k_y+\hat{k}k_z).(\hat{\imath}x+\hat{\jmath}y+\hat{k}z)=xk_x+yk_y+zk_z$. Comparing it with the given expression we get $xk_x+yk_y+zk_z=10^{+7}(\mathrm{x}+2\mathrm{y}+3\mathrm{z})$ Thereby $\Rightarrow k_x=10^7,\ k_y=2\times10^7\ \&\ k_z=3\times10^7$ or the vector $\vec{K}=(\hat{\imath}+2\hat{\jmath}+3\hat{k})\times10^7$

$\Rightarrow K=10^7\sqrt{14}$ Also the speed of the wave $c=\frac{\beta}{K}$ $\therefore$ $\beta=c\times10^7\sqrt{14}=3\times10^{15}\sqrt{14}$

Furtherfor any electromagnetic wave $\vec{k}\,.\,\vec{E}=0$ Therefore$10^{+7}(\hat{\imath}+2\hat{\jmath}+3\hat{k}).(3\hat{\jmath}+b\hat{k})$ $\times10^{-3}=0 \Rightarrow 2\times3+3b=0 \Rightarrow b=-2$ this makes option b wrong. Further the energy of an em wave is $=\frac{1}{2}\varepsilon_0E^2=\frac{1}{2}\varepsilon_0\left(\sqrt{3^2+2^2}\right)^2\times10^{-6}=6.5\varepsilon_0\,\mu J$. The magnetic field can be obtained as $B=\frac{E}{c}=\frac{10^{-3}\sqrt{13}}{3\times10^8}=1.20\times10^{-11}\,Tesla$ **Ans: a,c,d**

28. Snell's law is $\mu = \frac{\sin i}{\sin r} \Rightarrow \sqrt{2} = \frac{\sin 45}{\sin r} \Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30^0$

The critical angle is $\sin^{-1}\frac{1}{\sqrt{2}} = 45^0$.



For the emergence, theangle of incidence at curved surface must be less than 45^0 therefore angle θ should be greater than

$$\theta_{\min} = 180 - \left(90 - \sin^{-1}\left(\frac{\sin i}{\mu}\right)\right) - \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\theta_{\min} = [180 - (90 - 30) - 45] = 75^0$$

Towards the upper edge angle θ must be less than

$$\theta_{\max} = \left(90 + \sin^{-1}\left(\frac{\sin i}{\mu}\right)\right) + \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\theta_{\max} = (90 + 30) + 45 = 165$$

Thus we obtain $\theta_{\min} < \theta < \theta_{\max}$ *as* $75^0 \le \theta \le 165^0$ for the emergence of light through the curved surface.Thus the light will come out only for the angle θ lying within the range $\theta_{\max} - \theta_{\min} = 2\sin^{-1}\left(\frac{1}{\mu}\right)$ which is independent of the angle of incidence but depends on the refractive index (μ) of the material. Of course the range is same for all values of angle of incidence yet the values of $\theta_{\min}$ *and* $\theta_{\max}$ are different for different values of angle of incidence. Hence option **b is not correct**. The light coming out of the curved surface will go away from normal hence towards the line which is the increased radius for $\theta = 120^0$ and thus form a convergent beam towards the enhanced radius corresponding to $\theta = 120^0$.

Ans: a, c, d

29. The rate of flow of heat in a solid rod is expressed as the thermal current

$H = \frac{dQ}{dt} = KA\left(-\frac{dT}{dx}\right)$ Given that the thermal conductivity $K = \frac{\alpha}{T}$ so one can write

$H\int_0^l dx = -\alpha A\int_{90}^{10}\frac{dT}{T} \Rightarrow Hl = -\alpha A \ln\frac{10}{90} = 2\alpha A \ln 3$ using $l = 2m$ we get

$H = \alpha A \ln 3 = 1.1\alpha A$ Further at any intermediate location at a distance x from hot end

$H\int_0^x dx = -\alpha A\int_{90}^{T}\frac{dT}{T} \Rightarrow Hx = -\alpha A \ln\frac{T}{90} \Rightarrow T = 90 \times e^{-Hx/\alpha A}$

$At\ x = 0.5\ m,\ T = 90 \times e^{-Hx/\alpha A} = 90e^{-1.1 \times 0.5} = 51.96^0\ C$ Also

$At\ x = 1.5m,\ T = 90e^{-1.1 \times 1.5} = 17.32^0 C$ The temperature gradient may be expressed as

$\frac{dT}{dx} = -\frac{TH}{\alpha A}$ is higher near the hotter end than that near the colder end.

Ans: a, b, c, d

30. According to Bohr theory, in an hydrogen atom an electron revolves round the proton, the centripetal force being provided by electrostatic attraction. Such that

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e.e}{r^2} \quad \text{or}\; mv^2 r = Ke^2 ---(1)$$

where $v\, and\, r$ are the velocity of electron and the radius of the orbit and $K = \frac{1}{4\pi\varepsilon_0}$

According to Bohr quantum condition (second postulate)

$$mvr = n\frac{h}{2\pi} \;\; or \;\; mvr = n\hbar ---(2)$$

Dividing eq (1) by eq (2) $v = \frac{Ke^2}{n\hbar} \Longrightarrow$ v does not depend on mass. Now substituting v in equation (2)

$$m\left(\frac{Ke^2}{n\hbar}\right)r = n\hbar \Rightarrow r = \frac{n^2\hbar^2}{Kme^2} \text{ showing that } r \propto \frac{1}{m}$$

The total energy E = KE + PE

$$= \frac{1}{2}mv^2 - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = \frac{1}{2}m\left(\frac{Ke^2}{n\hbar}\right)^2 - \frac{Ke^2}{r}$$

$$E = \frac{1}{2}\frac{mK^2e^4}{n^2\hbar^2} - \frac{Ke^2}{n^2\hbar^2}Kme^2 \;\; E_n = -\frac{1}{2}\frac{mK^2e^4}{n^2\hbar^2} \Rightarrow E \propto m$$

To understand the situation more specifically, one replaces the mass of electron by the effective mass i.e. the reduced mass (μ) which for the case of hydrogen atom is

$\mu = \frac{m \times 1836m}{m + 1836m} \approx m(\text{electron mass})$ Thus for hydrogen atom the

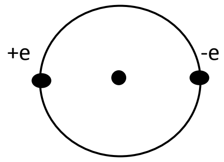
Radius of first orbit $r = a_0 = \frac{\hbar^2}{Kme^2}$ we have substituted n = 1

Velocity of electron in first orbit $v = v_0 = \frac{Ke^2}{\hbar}$

Energy of electron in first orbit E= $E_0 = -\frac{1}{2}\frac{mK^2e^4}{\hbar^2} \Rightarrow E \propto m$

A positronium is a short lived atomic entity in which a negatively charged electron is said to revolve round a positron (a positive particle having charge and mass equal to an electron even sometimes known as a positive electron) Since the particles have equal mass, the rotation takes place around the centre of mass which lies midway between the two.

In case of positronium the reduced mass is $\mu = \frac{mm}{m+m} = \frac{m}{2}$



Thereby the radius becomes $r = a = \frac{\hbar^2}{K(m/2)e^2} = \frac{2\hbar^2}{Kme^2} = 2a_0$

And energy becomes $E = -\frac{1}{2}\frac{(m/2)K^2e^4}{n^2\hbar^2} \Rightarrow E = -\frac{1}{4}\frac{mK^2e^4}{n^2\hbar^2} = \frac{E_0}{2}$ **Ans: a, c**

31. The focal length f_2 of a lens of refractive index μ and radii of curvature R_1 and R_2 when the refractive index of the object space is μ_1 and that of image space is μ_2 is calculated by

$\frac{\mu_2}{f_2} = \frac{\mu - \mu_1}{R_1} - \frac{\mu - \mu_2}{R_2}$ If the lens is kept in air $\mu_1 = 1$ and $\mu_2 = 1$ then

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{-60}\right) = \frac{1}{30} \Rightarrow f = 30\,cm$ Hence a is correct.

When the lens is silvered on the surface of radius 60 cm, it will behave as a concave mirror of focal length f_M such that

$\frac{1}{f_M} = \sum \frac{1}{f} = \frac{1}{f_{lens}} + \frac{2}{R} + \frac{1}{f_{lens}} = \frac{1}{30} + \frac{2}{60} + \frac{1}{30} = \frac{6}{60} = \frac{1}{10}$ *means* $f_M = 10\,cm$

Hence the option b is correct.

When the image space is filled with a liquid of refractive index $\mu_2 = \frac{5}{3}$, the object space still being air ($\mu_1 = 1$), the second focal length of the lens is obtained by $\frac{5}{3f_2} = \frac{1.5 - 1}{+20} - \frac{1.5 - \frac{5}{3}}{-60}$

$\Rightarrow f_2 = +\,75\,cm$ Also the first focal length in this case is $f_1 = +\,45\,cm$ so the lens still behaves as convex lens and not a concave (diverging) lens. Hence option c is wrong.

Considering a different situation when air in object space and water $\left(\mu = \frac{4}{3}\right)$ in image space, the second focal length of lens then is $\frac{4}{3f_2} = \frac{1.5 - 1}{20} - \frac{1.5 - 4/3}{-60} = \frac{1}{36} \Rightarrow f_2 = +\,48\,cm$

Hence a beam of light incident parallel to the principal axis focuses 48 cm behind the lens. Hence option d is correct. **Ans: a, b, d**

32. A poorly conducting thick hollow cylinder is placed coaxially inside a long solenoid. If we consider a circle of radius r (a < r < b), the magnetic flux through this area shall be $\phi = \pi r^2 \beta t$

The induced emf therefore shall be $\varepsilon = -\frac{d\phi}{dt} = -\,\pi r^2 \beta$ Thus $|\varepsilon| = \pi r^2 \beta$ If R be the resistance offered to the circulating current then $\frac{1}{R} = \int_a^b \frac{hdr}{\rho \times 2\pi r} = \frac{h}{2\pi\rho} \ln\frac{b}{a}$ Thereby

$R = \frac{2\pi\rho}{h \times \ln\frac{b}{a}}$ Further the circulating current induced in the thick hollow poorly conducting cylinder is $i = \frac{\varepsilon}{R} = \int_a^b \pi r^2 \beta \frac{hdr}{\rho 2\pi r} = \frac{\beta h}{2\rho}\int_a^b r dr = \frac{\beta h}{4\rho}\left(b^2 - a^2\right)$

The time varying magnetic field parallel to the axis of the solenoid produces an electric field even outside the solenoid. The lines of force being circular with their centres lying on the axis of the solenoid so option d is wrong. **Ans: a, b, c**